Chapter 3

Motion In Two Dimension

The motion of an object is called two dimensional, if two of the three co-ordinates required to specify the position of the object in space, change *w.r.t* time.

In such a motion, the object moves in a plane. For example, a billiard ball moving over the billiard table, an insect crawling over the floor of a room, earth revolving around the sun *etc.*

Two special cases of motion in two dimension are

1. Projectile motion 2. Circular motion

Introduction of Projectile Motion

A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, he will be safe but at the instant the bullet leaves the barrel of gun, if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path.

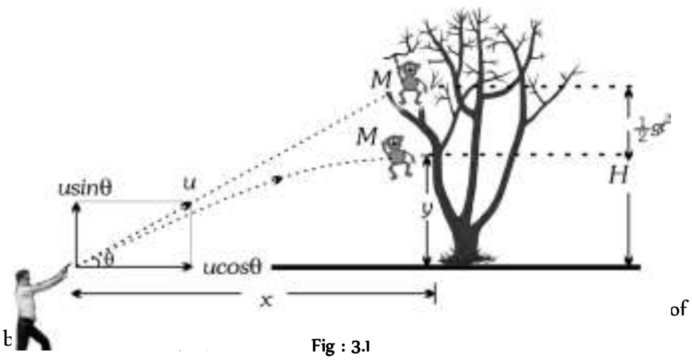


Fig : 3.1

If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.

Projectile

A body which is in flight through the atmosphere under the effect of gravity alone and is not being propelled by any fuel is called projectile.

Example:

(i) A bomb released from an aeroplane in level flight

(ii) A bullet fired from a gun

(iii) An arrow released from bow

(iv) A Javelin thrown by an athlete

Assumptions of Projectile Motion

(1) There is no resistance due to air.

(2) The effect due to curvature of earth is negligible.

(3) The effect due to rotation of earth is negligible.

(4) For all points of the trajectory, the acceleration due to gravity '*g*' is constant in magnitude and direction.

Principle of Physical Independence of Motions

(1) The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts. Horizontal motion and vertical motion. These two motions take place independent of each other. This is called the principle of physical independence of motions.

(2) The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.

(3) The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.

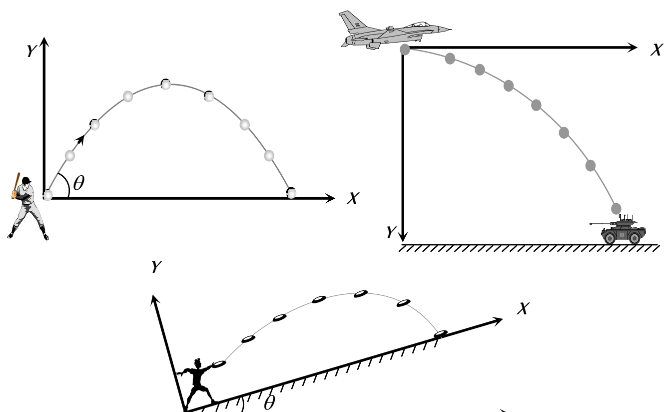
(4) The horizontal motion is a uniform motion and the vertical motion is a uniformly accelerated or retarded motion.

Types of Projectile Motion

(1) Oblique projectile motion

(2) Horizontal projectile motion

(3) Projectile motion on an inclined plane



Oblique Projectile

In projectile motion, horizontal component of velocity (u cosθ), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity (u sin θ), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

(1) **Equation of trajectory :** A projectile is thrown with velocity u at an angle θ with the horizontal. The velocity u can be resolved into two rectangular components.

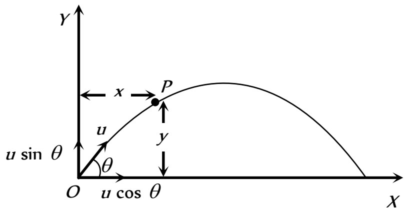


Fig : 3.3

v cos θ component along X–axis and u sin θ component along Y–axis.

For horizontal motion $x = u\cos\theta \times t \Rightarrow t = \dfrac{x}{u\cos\theta}$ ... (i)

For vertical motion $y = (u\sin\theta)t - \dfrac{1}{2}gt^2$ ...(ii)

From equation (i) and (ii)

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

This equation shows that the trajectory of projectile is parabolic because it is similar to equation of parabola

$y = ax - bx$

Note : ❑ Equation of oblique projectile also can be written as

$$y = x\tan\theta\left[1 - \frac{x}{R}\right] \text{ (where } R = \text{horizontal range} = \frac{u^2\sin 2\theta}{g}\text{)}$$

(2) **Displacement of projectile ($\vec{r}$) :** Let the particle acquires a position P having the coordinates (x, y) just after time t from the instant of projection. The corresponding position vector of the particle at time t is $\vec{r}$ as shown in the figure.

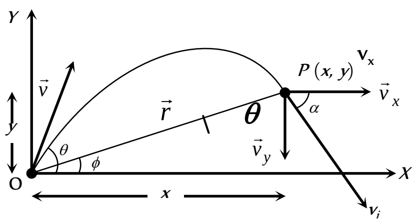


Fig : 3.4

$$\vec{r} = x\hat{i} + y\hat{j} \qquad ...(i)$$

The horizontal distance covered during time t is given as

$$x = v_x t \Rightarrow x = u\cos\theta\, t \qquad ...(ii)$$

The vertical velocity of the particle at time t is given as

$$v_y = (v_0)_y - gt, \qquad ...(iii)$$

Now the vertical displacement y is given as

$$y = u\sin\theta\, t - 1/2\, gt^2 \qquad ...(iv)$$

Putting the values of x and y from equation (ii) and equation (iv) in equation (i) we obtain the position vector at any time t as

$$\vec{r} = (u\cos\theta)t\,\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$

$$\Rightarrow r = \sqrt{(u\,t\cos\theta)^2 + \left((u\,t\sin\theta) - \frac{1}{2}gt^2\right)^2}$$

$$r = u\,t\sqrt{1 + \left(\frac{gt}{2u}\right)^2 - \frac{gt\sin\theta}{u}} \text{ and } \phi = \tan^{-1}(y/x)$$

$$= \tan^{-1}\left(\frac{ut\sin\theta - \frac{1}{2}gt^2}{(u\,t\cos\theta)}\right) \text{ or } \phi = \tan^{-1}\left(\frac{2u\sin\theta - gt}{2u\cos\theta}\right)$$

Note : ❑ The angle of elevation ϕ of the highest point of the projectile and the angle of projection θ are related to each other as

$$\tan\phi = \frac{1}{2}\tan\theta$$

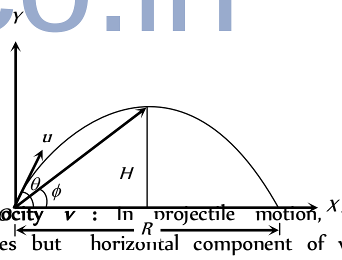


Fig : 3.5

(3) **Instantaneous velocity v :** In projectile motion, vertical component of velocity changes but horizontal component of velocity remains always constant.

Example : When a man jumps over the hurdle leaving behind its skateboard then vertical component of his velocity is changing, but not the horizontal component which matches with the skateboard velocity.

As a result, the skateboard stays underneath him, allowing him to land on it.

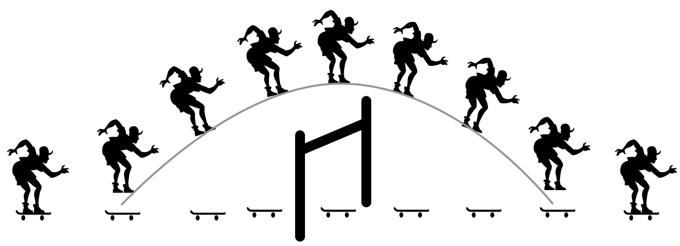


Fig : 3.6

Let v_i be the instantaneous velocity of projectile at time t, direction of this velocity is along the tangent to the trajectory at point P.

$$\vec{v}_i = v_x i + v_y\hat{j} \Rightarrow v_i = \sqrt{v_x^2 + v_y^2}$$

$= \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$

$v_i = \sqrt{u^2 + g^2t^2 - 2u\,gt\sin\theta}$

Direction of instantaneous velocity $\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$

or $\alpha = \tan^{-1}\left[\tan\theta - \frac{gt}{u}\sec\theta\right]$

(4) **Change in velocity :** Initial velocity (at projection point) $\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Final velocity (at highest point) $\vec{u}_f = u\cos\theta\,\hat{i} + 0\,\hat{j}$

(i) Change in velocity (Between projection point and highest point) $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -u\sin\theta\,\hat{j}$

When body reaches the ground after completing its motion then final velocity $\vec{u}_f = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

(ii) Change in velocity (Between complete projectile motion) $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -2u\sin\theta\,\hat{i}$

(5) **Change in momentum :** Simply by the multiplication of mass in the above expression of velocity (Article-4).

(i) Change in momentum (Between projection point and highest point) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -mu\sin\theta\,\hat{j}$

(ii) Change in momentum (For the complete projectile motion) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -2mu\sin\theta\,\hat{j}$

(6) **Angular momentum :** Angular momentum of projectile at highest point of trajectory about the point of projection is given by

$$L = mvr \qquad \left[\text{Here } r = H = \frac{u^2\sin^2\theta}{2g}\right]$$

$$\therefore \quad L = m\,u\cos\theta\,\frac{u^2\sin^2\theta}{2g} = \frac{m\,u^3\cos\theta\sin^2\theta}{2g}$$

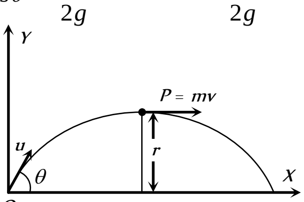


Fig : 3.7

(7) **Time of flight :** The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.

For vertical upward motion $0 = u\sin\theta - gt$

$\Rightarrow t = (u\sin\theta/g)$

Now as time taken to go up is equal to the time taken to come down so

Time of flight $T = 2t = \frac{2u\sin\theta}{g}$

(i) Time of flight can also be expressed as : $T = \frac{2.u_y}{g}$ (where u_y is the vertical component of initial velocity).

(ii) For complementary angles of projection θ and $90° - \theta$

(a) Ratio of time of flight $= \frac{T_1}{T_2} = \frac{2u\sin\theta/g}{2u\sin(90-\theta)/g}$

$= \tan\theta \Rightarrow \frac{T_1}{T_2} = \tan\theta$

(b) Multiplication of time of flight $= T_1T_2 = \frac{2u\sin\theta}{g}\frac{2u\cos\theta}{g}$

$\Rightarrow T_1T_2 = \frac{2R}{g}$

(iii) If t_1 is the time taken by projectile to rise upto point p and t_2 is the time taken in falling from point p to ground level then

$t_1 + t_2 = \frac{2u\sin\theta}{g}$ = time of flight or $u\sin\theta = \frac{g(t_1+t_2)}{2}$

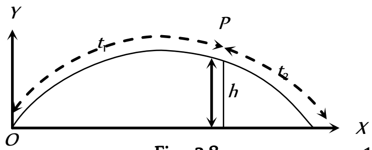


Fig : 3.8

and height of the point p is given by $h = u\sin\theta\,t_1 - \frac{1}{2}gt_1^2$

$h = g\frac{(t_1+t_2)}{2}t_1 - \frac{1}{2}gt_1^2$

by solving $h = \frac{g\,t_1t_2}{2}$

(iv) If B and C are at the same level on trajectory and the time difference between these two points is t_1, similarly A and D are also at the same level and the time difference between these two positions is t_2 then

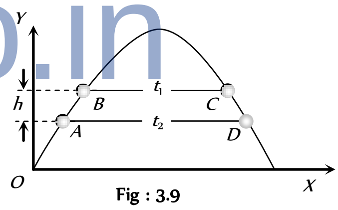


Fig : 3.9

$t_2^2 - t_1^2 = \frac{8h}{g}$

(8) **Horizontal range :** It is the horizontal distance travelled by a body during the time of flight.

So by using second equation of motion in x-direction

$R = u\cos\theta \times T$

$= u\cos\theta \times (2u\sin\theta/g)$

$= \frac{u^2\sin 2\theta}{g}$

$R = \frac{u^2\sin 2\theta}{g}$

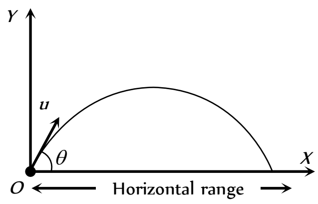


Fig : 3.10

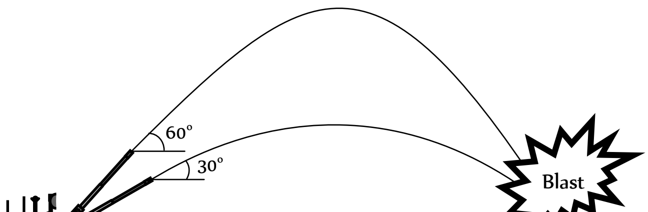
(i) Range of projectile can also be expressed as :

$R = u\cos\theta \times T = u\cos\theta\,\frac{2u\sin\theta}{g}$

$= \frac{2u\cos\theta\,u\sin\theta}{g} = \frac{2u_xu_y}{g}$

$\therefore R = \frac{2u_xu_y}{g}$ (where u_x and u_y are the horizontal and vertical component of initial velocity)

(ii) If angle of projection is changed from θ to $\theta' = (90 - \theta)$ then range remains unchanged.



$$R' = \frac{u^2 \sin 2\theta'}{g} = \frac{u^2 \sin[2(90^o - \theta)]}{g} = \frac{u^2 \sin 2\theta}{g} = R$$

So a projectile has same range at angles of projection θ and $(90 - \theta)$, though time of flight, maximum height and trajectories are different.

These angles θ and $90° - \theta$ are called complementary angles of projection and for complementary angles of projection, ratio of range

$$\frac{R_1}{R_2} = \frac{u^2 \sin 2\theta / g}{u^2 \sin[2(90^o - \theta)]/g} = 1 \Rightarrow \frac{R_1}{R_2} = 1$$

(iii) For angle of projection $\theta_1 = (45 - \alpha)$ and $\theta_2 = (45 + \alpha)$, range will be same and equal to $u^2 \cos 2\alpha/g$.

θ_1 and θ_2 are also the complementary angles.

(iv) Maximum range : For range to be maximum

$$\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{u^2 \sin 2\theta}{g}\right] = 0$$

$\Rightarrow \cos 2\theta = 0$ *i.e.* $2\theta = 90°$ $\Rightarrow$ $\theta = 45°$

and $R_{max} = (u^2/g)$

i.e., a projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g).

When the range is maximum, the height H reached by the projectile

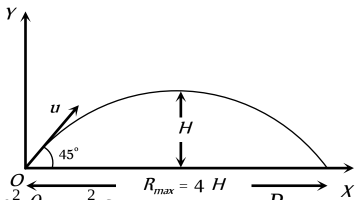


Fig : 3.12

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

i.e., if a person can throw a projectile to a maximum distance R_{max}, The maximum height during the flight to which it will rise is $\left(\frac{R_{max}}{4}\right)$.

(v) Relation between horizontal range and maximum height : $R = \frac{u^2 \sin 2\theta}{g}$ and $H = \frac{u^2 \sin^2 \theta}{2g}$

$$\therefore \frac{R}{H} = \frac{u^2 \sin 2\theta / g}{u^2 \sin^2 \theta / 2g} = 4 \cot\theta \Rightarrow R = 4H \cot\theta$$

(vi) If in case of projectile motion range R is n times the maximum height H

i.e. $R = nH \Rightarrow \frac{u^2 \sin 2\theta}{g} = n\frac{u^2 \sin^2 \theta}{2g}$

$\Rightarrow \tan\theta = [4/n]$ or $\theta = \tan^{-1}[4/n]$

The angle of projection is given by $\theta = \tan^{-1}[4/n]$

Note : If $R = H$ then $\theta = \tan^{-1}(4)$ or $\theta = 76^o$.

If $R = 4H$ then $\theta = \tan^{-1}(1)$ or $\theta = 45^o$.

(9) **Maximum height :** It is the maximum height from the point of projection, a projectile can reach.

So, by using $v^2 = u^2 + 2as$

$$0 = (u \sin\theta)^2 - 2gH$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

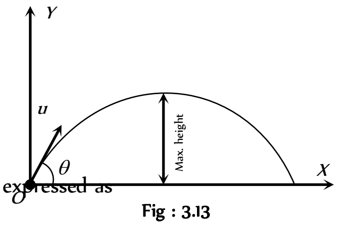


Fig : 3.13

(i) Maximum height can also be expressed as

$H = \frac{u_y^2}{2g}$ (where u_y is the vertical component of initial velocity).

(ii) $H_{max} = \frac{u^2}{2g}$ (when $\sin\theta = max = 1$ *i.e.,* $\theta = 90°$)

i.e., for maximum height body should be projected vertically upward. So it falls back to the point of projection after reaching the maximum height.

(iii) For complementary angles of projection θ and $90° - \theta$

Ratio of maximum height

$$\frac{H_1}{H_2} = \frac{u^2 \sin^2 \theta / 2g}{u^2 \sin^2 (90^o - \theta)/2g} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

$$\therefore \frac{H_1}{H_2} = \tan^2 \theta$$

(10) **Projectile passing through two different points on same height at time t_1 and t_2 :** If the particle passes two points situated at equal height y at $t = t_1$ and $t = t_2$, then

(i) **Height (y):** $y = (u \sin\theta) t_1 - \frac{1}{2} g t_1^2$ ...(i)

and $y = (u \sin\theta) t_2 - \frac{1}{2} g t_2^2$ ...(ii)

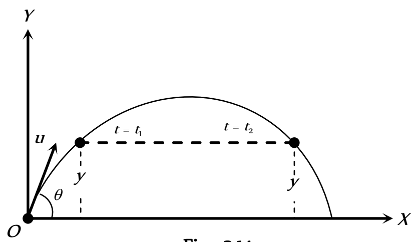


Fig : 3.14

Comparing equation (i) with equation (ii)

$$u \sin\theta = \frac{g(t_1 + t_2)}{2}$$

Substituting this value in equation (i)

$$y = g\left(\frac{t_1 + t_2}{2}\right) t_1 - \frac{1}{2} g t_1^2 \Rightarrow y = \frac{g t_1 t_2}{2}$$

(ii) **Time (t_1 and t_2):** $y = u \sin\theta \, t - \frac{1}{2} g t^2$

$$t^2 - \frac{2u\sin\theta}{g}t + \frac{2y}{g} = 0 \Rightarrow t = \frac{u\sin\theta}{g}\left[1 \pm \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

$$t_1 = \frac{u\sin\theta}{g}\left[1 + \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

and $t_2 = \frac{u\sin\theta}{g}\left[1 - \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$

(11) **Motion of a projectile as observed from another projectile :** Suppose two balls A and B are projected simultaneously from the origin, with initial velocities u_1 and u_2 at angle θ_1 and θ_2 respectively with the horizontal.

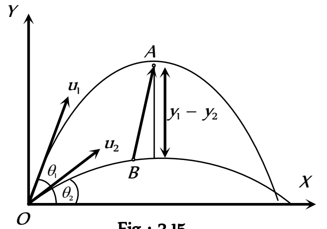


Fig : 3.15

The instantaneous positions of the two balls are given by

Ball A : $x_1 = (u_1\cos\theta_1)t,\ y_1 = (u_1\sin\theta_1)t - \frac{1}{2}gt^2$

Ball B : $x_2 = (u_2\cos\theta_2)t,\ y_2 = (u_2\sin\theta_2)t - \frac{1}{2}gt^2$

The position of the ball A with respect to ball B is given by

$$x = x_1 - x_2 = (u_1\cos\theta_1 - u_2\cos\theta_2)t$$

$$y = y_1 - y_2 = (u_1\sin\theta_1 - u_2\sin\theta_2)t$$

Now $\frac{y}{x} = \left(\frac{u_1\sin\theta_1 - u_2\sin\theta_2}{u_1\cos\theta_1 - u_2\cos\theta_2}\right) =$ constant

Thus motion of a projectile relative to another projectile is a straight line.

(12) **Energy of projectile :** When a projectile moves upward its kinetic energy decreases, potential energy increases but the total energy always remain constant.

If a body is projected with initial kinetic energy $K(=1/2\ mu^2)$, with angle of projection θ with the horizontal then at the highest point of trajectory

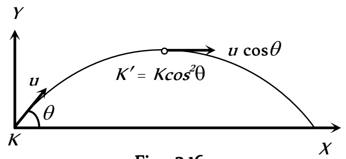


Fig : 3.16

(i) **Kinetic energy**

$$= \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta$$

$$\therefore\ K' = K\cos^2\theta$$

(ii) **Potential energy** $= mgH = mg\frac{u^2\sin^2\theta}{2g}$

$$= \frac{1}{2}mu^2\sin^2\theta \qquad \left(\text{As } H = \frac{u^2\sin^2\theta}{2g}\right)$$

$$= K\sin^2\theta$$

(iii) **Total energy** = Kinetic energy + Potential energy

$$= \frac{1}{2}mu^2\cos^2\theta + \frac{1}{2}mu^2\sin^2\theta$$

$= \frac{1}{2}mu^2 =$ Energy at the point of projection.

This is in accordance with the law of conservation of energy.

Horizontal Projectile

When a body is projected horizontally from a certain height 'y' vertically above the ground with initial velocity u. If friction is considered to be absent, then there is no other horizontal force which can affect the horizontal motion. The horizontal velocity therefore remains constant and so the object covers equal distance in horizontal direction in equal intervals of time.

The horizontal velocity therefore remains constant and so the object covers equal distance in horizontal direction in equal intervals of time.

(1) **Trajectory of horizontal projectile :** The horizontal displacement x is governed by the equation

$x = ut \Rightarrow t = \frac{x}{u}$ .... (i)

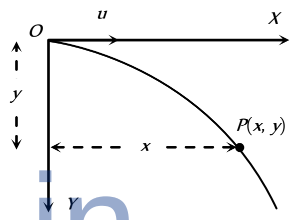


Fig : 3.17

The vertical displacement y is governed by

$y = \frac{1}{2}gt^2$ .... (ii)

(since initial vertical velocity is zero)

By substituting the value of t in equation (ii) $y = \frac{1}{2}\frac{gx^2}{u^2}$

(2) **Displacement of Projectile** $(\vec{r})$: After time t, horizontal displacement $x = ut$ and vertical displacement $y = \frac{1}{2}gt^2$.

So, the position vector $\vec{r} = ut\,\hat{i} + \frac{1}{2}gt^2\,\hat{j}$

Therefore $r = ut\sqrt{1 + \left(\frac{gt}{2u}\right)^2}$ and $\alpha = \tan^{-1}\left(\frac{gt}{2u}\right)$

$\alpha = \tan^{-1}\left(\sqrt{\frac{gy}{2}}\Big/u\right)$ $\left(\text{as } t = \sqrt{\frac{2y}{g}}\right)$

(3) **Instantaneous velocity :** Throughout the motion, the horizontal component of the velocity is $v_x = u$.

The vertical component of velocity increases with time and is given by

$v_y = 0 + g\,t = g\,t$ (From $v = u + g\,t$)

So, $\vec{v} = v_x\hat{i} + v_y\hat{j} = u\,\hat{i} + g\,t\,\hat{j}$

i.e. $v = \sqrt{u^2 + (gt)^2} \;\; = u\sqrt{1 + \left(\frac{gt}{u}\right)^2}$

Again $\vec{v} = u\hat{i} + \sqrt{2gy}\,\hat{j}$

i.e. $v = \sqrt{u^2 + 2gy}$

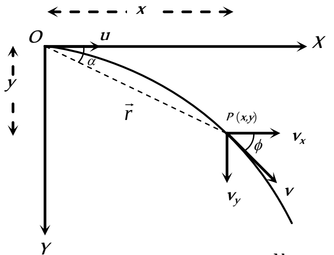


Fig : 3.18

Direction of instantaneous velocity : $\tan\phi = \frac{v_y}{v_x}$

$\Rightarrow \;\; \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) \; = \tan^{-1}\left(\frac{\sqrt{2gy}}{u}\right)$ or $\phi = \tan^{-1}\left(\frac{gt}{u}\right)$

Where ϕ is the angle of instantaneous velocity from the horizontal.

(4) **Time of flight :** If a body is projected horizontally from a height h with velocity u and time taken by the body to reach the ground is T, then

$h = 0 + \frac{1}{2}gT^2$ (for vertical motion)

$T = \sqrt{\frac{2h}{g}}$

(5) **Horizontal range :** Let R is the horizontal distance travelled by the body

$R = uT + \frac{1}{2}\,0\,T^2$ (for horizontal motion)

$R = u\sqrt{\frac{2h}{g}}$

(6) If projectiles A and B are projected horizontally with different initial velocity from same height and third particle C is dropped from same point then

(i) All three particles will take equal time to reach the ground.

(ii) Their net velocity would be different but all three particle possess same vertical component of velocity.

(iii) The trajectory of projectiles A and B will be straight line *w.r.t.* particle C.

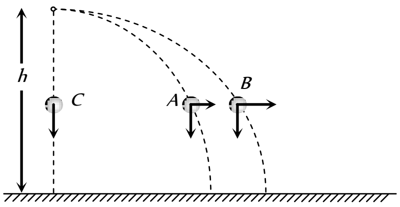


Fig : 3.19

(7) If various particles thrown with same initial velocity but in different direction then

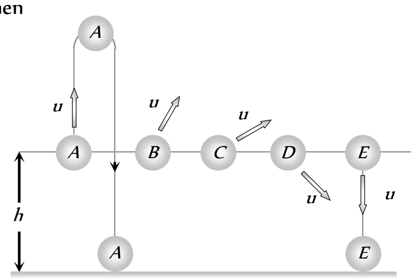


Fig : 3.20

(i) They strike the ground with same speed at different times irrespective of their initial direction of velocities.

(ii) Time would be least for particle E which was thrown vertically downward.

(iii) Time would be maximum for particle A which was thrown vertically upward.

Projectile Motion on An Inclined Plane

Let a particle be projected up with a speed u from an inclined plane which makes an angle α with the horizontal and velocity of projection makes an angle θ with the inclined plane.

We have taken reference *x*-axis in the direction of plane.

Hence the component of initial velocity parallel and perpendicular to the plane are equal to $u\cos\theta$ and $u\sin\theta$ respectively *i.e.* $u_{\parallel} = u\cos\theta$ and $u_{\perp} = u\sin\theta$.

The component of g along the plane is $g\sin\alpha$ and perpendicular to the plane is $g\cos\alpha$ as shown in the figure *i.e.* $a_{\parallel} = -g\sin\alpha$ and $a_{\perp} = g\cos\alpha$.

Therefore the particle decelerates at a rate of $g\sin\alpha$ as it moves from O to P.

(1) **Time of flight :** We know for oblique projectile motion $T = \frac{2u\sin\theta}{g}$ or we can say $T = \frac{2u_{\perp}}{a_{\perp}}$

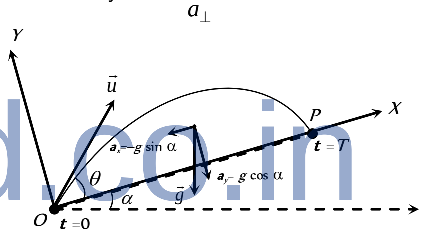


Fig : 3.21

$\therefore$ Time of flight on an inclined plane $T = \frac{2u\sin\theta}{g\cos\alpha}$

(2) **Maximum height :** We know for oblique projectile motion $H = \frac{u^2\sin^2\theta}{2g}$ or we can say $H = \frac{u_{\perp}^2}{2a_{\perp}}$

$\therefore$ Maximum height on an inclined plane $H = \frac{u^2\sin^2\theta}{2g\cos\alpha}$

(3) **Horizontal range :** For one dimensional motion $s = ut + \frac{1}{2}at^2$

Horizontal range on an inclined plane $R = u_{\parallel}T + \frac{1}{2}a_{\parallel}T^2$

$R = u\cos\theta\, T - \frac{1}{2}g\sin\alpha\, T^2$

$R = u\cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$

By solving $\quad R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$

(i) Maximum range occurs when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$

(ii) The maximum range along the inclined plane when the projectile is thrown upwards is given by

$$R_{max} = \frac{u^2}{g(1+\sin\alpha)}$$

(iii) The maximum range along the inclined plane when the projectile is thrown downwards is given by

$$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$$

Circular Motion

Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.

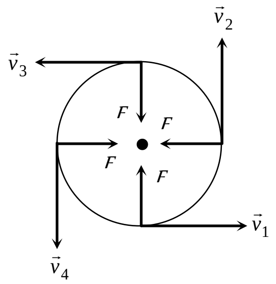


Fig : 3.22

Since this force is always at right angles to the displacement therefore no work is done by the force on the particle. Hence, its kinetic energy and thus speed is unaffected. But due to simultaneous action of the force and the velocity the particle follows resultant path, which in this case is a circle. Circular motion can be classified into two types – Uniform circular motion and non-uniform circular motion.

Variables of Circular Motion

(1) **Displacement and distance :** When particle moves in a circular path describing an angle θ during time t (as shown in the figure) from the position A to the position B, we see that the magnitude of the position vector $\vec{r}$ (that is equal to the radius of the circle) remains constant. *i.e.*, $|\vec{r}_1| = |\vec{r}_2| = r$ and the direction of the position vector changes from time to time.

(i) Displacement : The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position A to the position B is given by referring the figure.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \Rightarrow \Delta r = |\Delta\vec{r}| = |\vec{r}_2 - \vec{r}_1|$$

$$\Delta r = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\theta}$$

Putting $r_1 = r_2 = r$ we obtain

$$\Delta r = \sqrt{r^2 + r^2 - 2r.r\cos\theta}$$

$$\Rightarrow \Delta r = \sqrt{2r^2(1-\cos\theta)}$$

$$= \sqrt{2r^2\left(2\sin^2\frac{\theta}{2}\right)}$$

$$\Delta r = 2r\sin\frac{\theta}{2}$$

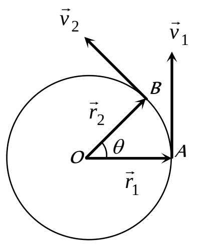


Fig : 3.23

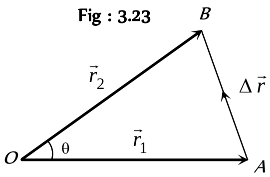


Fig : 3.24

(ii) Distance : The distanced covered by the particle during the time t is given as

d = length of the arc AB = $r\,\theta$

(iii) Ratio of distance and displacement : $\dfrac{d}{\Delta r} = \dfrac{r\theta}{2r\sin\theta/2}$

$$= \frac{\theta}{2}\mathrm{cosec}(\theta/2)$$

(2) **Angular displacement (θ) :** The angle turned by a body moving in a circle from some reference line is called angular displacement.

(i) Dimension = $[M^0L^0T^0]$ (as θ = arc/radius).

(ii) Units = Radian or Degree. It is some time also specified in terms of fraction or multiple of revolution.

(iii) $2\pi \text{ rad} = 360^o = 1 \text{ Revolution}$

(iv) Angular displacement is a axial vector quantity.

Its direction depends upon the sense of rotation of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.

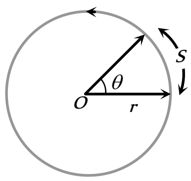


Fig : 3.25

(v) Relation between linear displacement and angular displacement $\vec{s} = \vec{\theta} \times \vec{r}$

or $s = r\theta$

(3) **Angular velocity (ω) :** Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

(i) Angular velocity $\omega = \dfrac{\text{angle traced}}{\text{time taken}} = \underset{\Delta t\to 0}{Lt}\dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

$$\therefore \omega = \frac{d\theta}{dt}$$

(ii) Dimension : $[M^0L^0T^{-1}]$

(iii) Units : Radians per second ($rad.s^{-1}$) or Degree per second.

(iv) Angular velocity is an axial vector.

Its direction is the same as that of $\Delta\theta$. For anticlockwise rotation of the point object on the circular path, the direction of ω, according to Right hand rule is along the axis of circular path directed upwards. For clockwise rotation of the point object on the circular path, the direction of ω is along the axis of circular path directed downwards.

(v) Relation between angular velocity and linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$

(vi) For uniform circular motion ω remains constant where as for non-uniform motion ω varies with respect to time.

Note : It is important to note that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the circular motion is taking place in a plane perpendicular to it.

(4) **Change in velocity :** We want to know the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion as it moves from A to B during time t as shown in figure. The change in velocity vector is given as

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

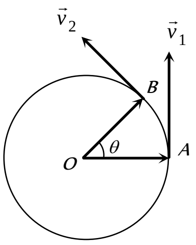


Fig : 3.26

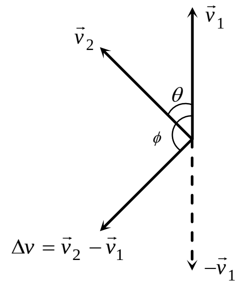


Fig : 3.27

or $|\Delta\vec{v}| = |\vec{v}_2 - \vec{v}_1|$ $\Rightarrow$ $\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos\theta}$

For uniform circular motion $v_1 = v_2 = v$

So $\Delta v = \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$

The direction of $\Delta\vec{v}$ is shown in figure that can be given as

$$\phi = \frac{180^o - \theta}{2} = (90^o - \theta/2)$$

(5) **Time period (T) :** In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.

(i) Units : second.

(ii) Dimension : $[M^0 L^0 T]$

(iii) Time period of second's hand of watch = 60 *second.*

(iv) Time period of minute's hand of watch = 60 *minute*

(v) Time period of hour's hand of watch = 12 *hour*

(6) **Frequency (n) :** In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.

(i) Units : s^{-1} or hertz (Hz).

(ii) Dimension : $[M^0 L^0 T^{-1}]$

Note : ❑ Relation between time period and frequency : If n is the frequency of revolution of an object in circular motion, then the object completes n revolutions in 1 second. Therefore, the object will complete one revolution in $1/n$ second.

$\therefore T = 1/n$

❑ Relation between angular velocity, frequency and time period : Consider a point object describing a uniform circular motion with frequency n and time period T. When the object completes one revolution, the angle traced at its axis of circular motion is 2π radians. It means, when time $t = T$, $\theta = 2\pi$ radians. Hence, angular velocity $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi n$ $\quad (\because T = 1/n)$

$$\omega = \frac{2\pi}{T} = 2\pi n$$

❑ If two particles are moving on same circle or different coplanar concentric circles in same direction with different uniform angular speeds ω_A and ω_B respectively, the angular velocity of B relative to A will be

$$\omega_{\text{rel}} = \omega_B - \omega_A$$

So the time taken by one to complete one revolution around O with respect to the other (i.e., time in which B complete one revolution around O with respect to the other (*i.e.*, time in which B completes one more or less revolution around O than A)

$$T = \frac{2\pi}{\omega_{\text{rel}}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2} \qquad \left[\text{as } T = \frac{2\pi}{\omega}\right]$$

Special case : If $\omega_B = \omega_A, \omega_{\text{rel}} = 0$ and so $T = \infty$., particles will maintain their position relative to each other. This is what actually happens in case of geostationary satellite $(\omega_1 = \omega_2 = \text{constant})$

(7) **Angular acceleration (α) :** Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) If $\Delta\omega$ be the change in angular velocity of the object in time interval Δt, while moving on a circular path, then angular acceleration of the object will be

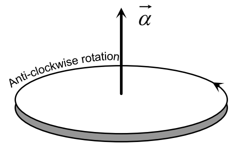


Fig : 3.28

$$\alpha = \underset{\Delta t \to 0}{Lt} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

(ii) Units : *rad. s*$^{-2}$

(iii) Dimension : $[M^0 L^0 T^{-2}]$

(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since ω is constant so $\alpha = \frac{d\omega}{dt} = 0$

(vi) For non-uniform circular motion $\alpha \neq 0$

Centripetal Acceleration

(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

(2) It always acts on the object along the radius towards the centre of the circular path.

(3) Magnitude of centripetal acceleration,

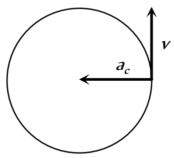


Fig : 3.29

$$a = \frac{v^2}{r} = \omega^2 r = 4\pi^2 n^2 r = \frac{4\pi^2}{T^2} r$$

(4) Direction of centripetal acceleration : It is always the same as that of $\Delta\vec{\upsilon}$. When Δt decreases, $\Delta\theta$ also decreases. Due to which $\Delta\vec{\upsilon}$ becomes more and more perpendicular to $\vec{\upsilon}$. When $\Delta t \to 0$, $\Delta\vec{\upsilon}$ becomes perpendicular to the velocity vector. As the velocity vector of the particle at an instant acts along the tangent to the circular path, therefore $\Delta\vec{\upsilon}$ and hence the centripetal acceleration vector acts along the radius of the circular path at that point and is directed towards the centre of the circular path.

Centripetal force

According to Newton's first law of motion, whenever a body moves in a straight line with uniform velocity, no force is required to maintain this velocity. But when a body moves along a circular path with uniform speed, its direction changes continuously *i.e.* velocity keeps on changing on account of a change in direction. According to Newton's second law of motion, a change in the direction of motion of the body can take place only if some external force acts on the body.

Due to inertia, at every point of the circular path; the body tends to move along the tangent to the circular path at that point (in figure). Since every body has directional inertia, a velocity cannot change by itself and as such we have to apply a force. But this force should be such that it changes the direction of velocity and not its magnitude. This is possible only if the force acts perpendicular to the direction of velocity. Because the velocity is along the tangent, this force must be along the radius (because the radius of a circle at any point is perpendicular to the tangent at that point). Further, as this force is to move the body in a circular path, it must acts towards the centre. This centre-seeking force is called the centripetal force.

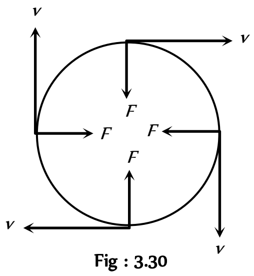


Fig : 3.30

Hence, centripetal force is that force which is required to move a body in a circular path with uniform speed. The force acts on the body along the radius and towards centre.

Formulae for centripetal force :

$$F = \frac{mv^2}{r} = m\omega^2 r = m\,4\pi^2 n^2 r = \frac{m4\pi^2 r}{T^2}$$

Table 3.1 : Centripetal force in different situation

Situation	Centripetal Force
A particle tied to a string and whirled in a horizontal circle	Tension in the string
Vehicle taking a turn on a level road	Frictional force exerted by the road on the tyres
A vehicle on a speed breaker	Weight of the body or a component of weight
Revolution of earth around the sun	Gravitational force exerted by the sun
Electron revolving around the nucleus in an atom	Coulomb attraction exerted by the protons in the nucleus
A charged particle describing a circular path in a magnetic field	Magnetic force exerted by the agent that sets up the magnetic field

Centrifugal Force

It is an imaginary force due to incorporated effects of inertia. When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer *A* who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer *B*, who is sharing the motion along the circular path (*i.e.*, the observer *B* is also rotating with the body with the same velocity), the body appears to be stationary before it is released. When the body is released, it appears to *B*, as if it has been thrown off along the radius away from the centre by some force. In reality no force is actually seen to act on the body. In absence of any real force the body tends to continue its motion in a straight line due to its inertia. The observer *A* easily relates this events to be due to inertia but since the inertia of both the observer *B* and the body is same, the observer *B* can not relate the above happening to inertia. When the centripetal force ceases to act on the body, the body leaves its circular path and continues to move in its straight-line motion but to observer *B* it appears that a real force has actually acted on the body and is responsible for throwing the body radially out-wards. This imaginary force is given a name to explain the effects of inertia to the observer who is sharing the circular motion of the body. This inertial force is called centrifugal force. Thus centrifugal force is a fictitious force which has significance only in a rotating frame of reference.

Work Done by Centripetal Force

The work done by centripetal force is always zero as it is perpendicular to velocity and hence instantaneous displacement.

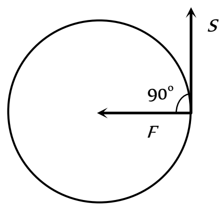


Fig : 3.31

Work done = Increment in kinetic energy of revolving body

Work done = 0

Also $W = \vec{F}.\vec{S} = F \cdot S\cos\theta = FS\cos 90^\circ = 0$

Example : (i) When an electron revolves around the nucleus in hydrogen atom in a particular orbit, it neither absorb nor emit any energy means its energy remains constant.

(ii) When a satellite established once in a orbit around the earth and it starts revolving with particular speed, then no fuel is required for its circular motion.

Skidding of Vehicle on A Level Road

When a vehicle takes a turn on a circular path it requires centripetal force.

If friction provides this centripetal force then vehicle can move in circular path safely if

Friction force ≥ Required centripetal force

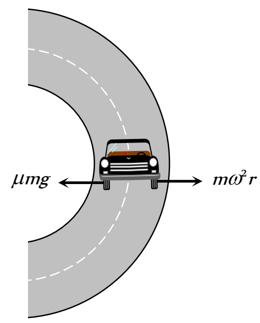


Fig : 3.32

$$\mu mg \geq \frac{mv^2}{r}$$

$$\therefore\ v_{safe} \leq \sqrt{\mu rg}$$

This is the maximum speed by which vehicle can take a turn on a circular path of radius *r*, where coefficient of friction between the road and tyre is μ.

Skidding of Object on A Rotating Platform

On a rotating platform, to avoid the skidding of an object (mass *m*) placed at a distance *r* from axis of rotation, the centripetal force should be provided by force of friction.

Centripetal force ≤ Force of friction

$m\omega r \leq \mu mg$

$$\therefore\ \omega_{max} = \sqrt{(\mu g / r)},$$

Hence maximum angular velocity of rotation of the platform is $\sqrt{(\mu g / r)}$, so that object will not skid on it.

Bending of A Cyclist

A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track, while going round a curve. Consider a cyclist of weight *mg* taking a turn of radius *r* with velocity *v*. In order to provide the necessary centripetal force, the cyclist leans through angle θ inwards as shown in figure.

The cyclist is under the action of the following forces :

The weight *mg* acting vertically downward at the centre of gravity of cycle and the cyclist.

The reaction *R* of the ground on cyclist. It will act along a line-making angle θ with the vertical.

The vertical component $R\cos\theta$ of the normal reaction *R* will balance the weight of the cyclist, while the horizontal component $R\sin\theta$ will provide the necessary centripetal force to the cyclist.

$$R\sin\theta = \frac{mv^2}{r} \quad \text{...(i)}$$

and $R\cos\theta = mg$ ...(ii)

Dividing equation (i) by (ii), we have

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2/r}{mg}$$

or $\tan\theta = \dfrac{v^2}{rg}$ ...(iii)

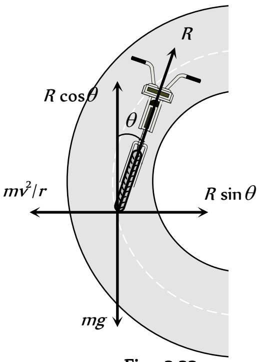


Fig : 3.33

Therefore, the cyclist should bend through an angle $\theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right)$

It follows that the angle through which cyclist should bend will be greater, if

(i) The radius of the curve is small *i.e.* the curve is sharper

(ii) The velocity of the cyclist is large.

Note : ❑ For the same reasons, an ice skater or an aeroplane has to bend inwards, while taking a turn.

Banking of A Road

For getting a centripetal force, cyclist bend towards the centre of circular path but it is not possible in case of four wheelers.

Therefore, outer bed of the road is raised so that a vehicle moving on it gets automatically inclined towards the centre.

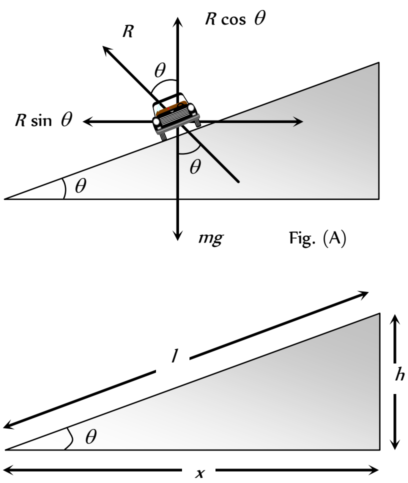


Fig : 3.34

In the figure (A) shown reaction *R* is resolved into two components, the component $R\cos\theta$ balances weight of vehicle

$\therefore\ R\cos\theta = mg$ ...(i)

and the horizontal component $R\sin\theta$ provides necessary centripetal force as it is directed towards centre of desired circle

Thus $R\sin\theta = \dfrac{mv^2}{r}$ ...(ii)

Dividing (ii) by (i), we have

$$\tan\theta = \frac{v^2}{rg} \quad \text{...(iii)}$$

or $\tan\theta = \dfrac{\omega^2 r}{g} = \dfrac{v\omega}{g}$ ...(iv) [As $v = r\omega$]

If l = width of the road, h = height of the outer edge from the ground level then from the figure (B)

$\tan\theta = \dfrac{h}{x} = \dfrac{h}{l}$ ...(v) [since θ is very small]

From equation (iii), (iv) and (v)

$$\tan\theta = \frac{v^2}{rg} = \frac{\omega^2 r}{g} = \frac{v\omega}{g} = \frac{h}{l}$$

Note : ❑ If friction is also present between the tyres and road then $\dfrac{v^2}{rg} = \dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}$

❑ Maximum safe speed on a banked frictional road $v = \sqrt{\dfrac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$

Overturning of Vehicle

When a car moves in a circular path with speed more than a certain maximum speed then it overturns even if friction is sufficient to avoid skidding and its inner wheel leaves the ground first

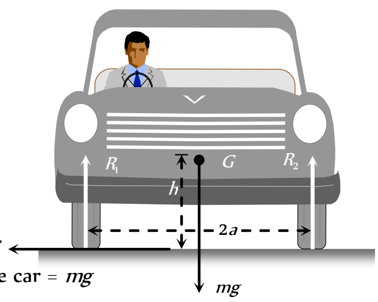


Fig : 3.35

Weight of the car = *mg*

Speed of the car = *v*

Radius of the circular path = *r*

Distance between the centre of wheels of the car = $2a$

Height of the centre of gravity (*G*) of the car from the road level = *h*

Reaction on the inner wheel of the car by the ground = R_1

Reaction on the outer wheel of the car by the ground = R_2

When a car move in a circular path, horizontal friction force *F* provides the required centripetal force i.e., $F = \dfrac{mv^2}{R}$ ...(i)

For rotational equilibrium, by taking the moment of forces R_1, R_2 and F about G

$$Fh + R_1 a = R_2 a \quad ...(ii)$$

As there is no vertical motion so $R_1 + R_2 = mg$ ...(iii)

By solving (i), (ii) and (iii)

$$R_1 = \frac{1}{2}M\left[g - \frac{v^2 h}{ra}\right] \quad ...(iv)$$

and $$R_2 = \frac{1}{2}M\left[g + \frac{v^2 h}{ra}\right] \quad ...(v)$$

It is clear from equation (iv) that if v increases value of R_1 decreases and for $R_1 = 0$

$$\frac{v^2 h}{ra} = g \text{ or } v = \sqrt{\frac{gra}{h}}$$

i.e. the maximum speed of a car without overturning on a flat road is given by $v = \sqrt{\dfrac{gra}{h}}$

Motion of Charged Particle In Magnetic Field

When a charged particle having mass m, charge q enters perpendicularly in a magnetic field B with velocity v then it describes a circular path.

Because magnetic force (qvB) works in the perpendicular direction of v and it provides required centripetal force

Magnetic force = Centripetal force

$$qvB = \frac{mv^2}{r}$$

$\therefore$ radius of the circular path

$$r = \frac{mv}{qB}$$

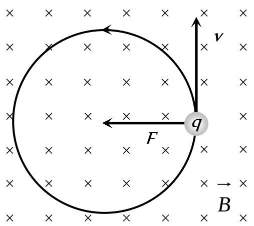


Fig : 3.36

Reaction of Road On Car

(1) When car moves on a concave bridge then

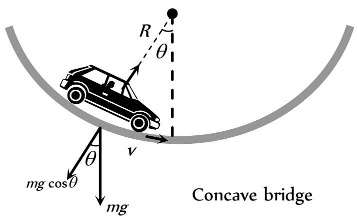


Fig : 3.37

Centripetal force = $R - mg\cos\theta = \dfrac{mv^2}{r}$

and reaction $R = mg\cos\theta + \dfrac{mv^2}{r}$

(2) When car moves on a convex bridge

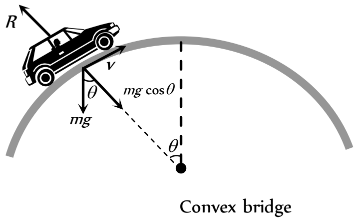


Fig : 3.38

Centripetal force = $mg\cos\theta - R = \dfrac{mv^2}{r}$

and reaction $R = mg\cos\theta - \dfrac{mv^2}{r}$

Non-Uniform Circular Motion

If the speed of the particle in a horizontal circular motion changes with respect to time, then its motion is said to be non-uniform circular motion.

Consider a particle describing a circular path of radius r with centre at O. Let at an instant the particle be at P and $\vec{\upsilon}$ be its linear velocity and $\vec{\omega}$ be its angular velocity.

Then, $\vec{\upsilon} = \vec{\omega} \times \vec{r}$ ...(i)

Differentiating both sides of w.r.t. time t we have

$$\frac{d\vec{\upsilon}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} \quad ...(ii)$$

Here, $\dfrac{d\vec{v}}{dt} = \vec{a}$, (Resultant acceleration)

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\upsilon}$$

$\dfrac{d\vec{\omega}}{dt} = \vec{\alpha}$ (Angular acceleration)

$$\vec{a} = \vec{a}_t + \vec{a}_c \quad ...(iii)$$

$\dfrac{d\vec{r}}{dt} = \vec{\upsilon}$ (Linear velocity)

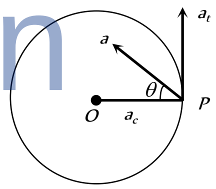


Fig : 3.39

Thus the resultant acceleration of the particle at P has two component accelerations

(1) **Tangential acceleration :** $\vec{a}_t = \vec{\alpha} \times \vec{r}$

It acts along the tangent to the circular path at P in the plane of circular path.

According to right hand rule since $\vec{\alpha}$ and $\vec{r}$ are perpendicular to each other, therefore, the magnitude of tangential acceleration is given by

$$|\vec{a}_t| = |\vec{\alpha} \times \vec{r}| = \alpha r \sin 90^o = \alpha r.$$

(2) **Centripetal (Radial) acceleration :** $\vec{a}_c = \vec{\omega} \times \vec{v}$

It is also called centripetal acceleration of the particle at P.

It acts along the radius of the particle at P.

According to right hand rule since $\vec{\omega}$ and $\vec{\upsilon}$ are perpendicular to each other, therefore, the magnitude of centripetal acceleration is given by

$$|\vec{a}_c| = |\vec{\omega} \times \vec{\upsilon}| = \omega\upsilon \sin 90^o = \omega\upsilon = \omega(\omega r) = \omega^2 r = \upsilon^2 / r$$

Table 3.2 : Tangential and centripetal acceleration

Centripetal acceleration	Tangential acceleration	Net acceleration	Type of motion
$a_c = 0$	$a_t = 0$	$a = 0$	Uniform

			translatory motion
$a_c = 0$	$a_t \neq 0$	$a = a_t$	Accelerated translatory motion
$a_c \neq 0$	$a_t = 0$	$a = a_c$	Uniform circular motion
$a_c \neq 0$	$a_t \neq 0$	$a = \sqrt{a_c^2 + a_t^2}$	Non-uniform circular motion

Note : ❑ Here a_t governs the magnitude of $\vec{v}$ while $\vec{a}_c$ its direction of motion.

(3) **Force** : In non-uniform circular motion the particle simultaneously possesses two forces

Centripetal force : $F_c = ma_c = \dfrac{mv^2}{r} = mr\omega^2$

Tangential force : $F_t = ma_t$

Net force : $F_{net} = ma = m\sqrt{a_c^2 + a_t^2}$

Note : ❑ In non-uniform circular motion work done by centripetal force will be zero since $\vec{F}_c \perp \vec{v}$

❑ In non uniform circular motion work done by tangential force will not be zero since $F_t \neq 0$

❑ Rate of work done by net force in non-uniform circular motion = rate of work done by tangential force

i.e. $P = \dfrac{dW}{dt} = \vec{F}_t.\vec{v}$

Equations of Circular Motion

For accelerated motion	For retarded motion
$\omega_2 = \omega_1 + \alpha t$	$\omega_2 = \omega_1 - \alpha t$
$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$	$\theta = \omega_1 t - \frac{1}{2}\alpha t^2$
$\omega_2^2 = \omega_1^2 + 2\alpha\theta$	$\omega_2^2 = \omega_1^2 - 2\alpha\theta$
$\theta_n = \omega_1 + \frac{\alpha}{2}(2n-1)$	$\theta_n = \omega_1 - \frac{\alpha}{2}(2n-1)$

Where

ω_1 = Initial angular velocity of particle

ω_2 = Final angular velocity of particle

α = Angular acceleration of particle

θ = Angle covered by the particle in time t

θ_n = Angle covered by the particle in n^{th} second

Motion in vertical circle

This is an example of non-uniform circular motion. In this motion body is under the influence of gravity of earth. When body moves from lowest point to highest point. Its speed decrease and becomes minimum at highest point. Total mechanical energy of the body remains conserved and *KE* converts into *PE* and vice versa.

(1) **Velocity at any point on vertical loop :** If u is the initial velocity imparted to body at lowest point then velocity of body at height h is given by

$$v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gl(1-\cos\theta)}$$

[As $h = l - l\cos\theta = l(1 - \cos\theta)$]

where l is the length of the string

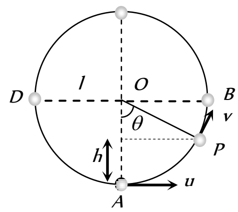


Fig : 3.40

(2) **Tension at any point on vertical loop :** Tension at general point *P*, According to Newton's second law of motion.

Net force towards centre = centripetal force

$$T - mg\cos\theta = \frac{mv^2}{l}$$

or $T = mg\cos\theta + \dfrac{mv^2}{l}$

$$T = \frac{m}{l}[u^2 - gl(2 - 3\cos\theta)]$$

[As $v = \sqrt{u^2 - 2gl(1-\cos\theta)}$]

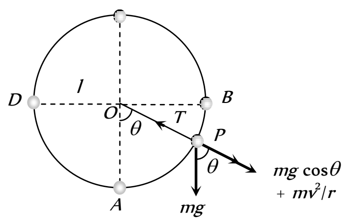


Fig : 3.41

Table 3.3 : Velocity and tension in a vertical loop

Position	Angle	Velocity	Tension
A	0°	u	$\frac{mu^2}{l} + mg$
B	90°	$\sqrt{u^2 - 2gl}$	$\frac{mu^2}{l} - 2mg$
C	180°	$\sqrt{u^2 - 4gl}$	$\frac{mu^2}{l} - 5mg$
D	270°	$\sqrt{u^2 - 2gl}$	$\frac{mu^2}{l} - 2mg$

It is clear from the table that : $T_A > T_B > T_C$ and $T_B = T_D$

$T_A - T_B = 3mg$,

$T_A - T_C = 6mg$

and $T_B - T_C = 3mg$

Table 3.4 : Various conditions for vertical motion

Velocity at lowest point	Condition
$u_A > \sqrt{5gl}$	Tension in the string will not be zero at any of the point and body will continue the circular motion.
$u_A = \sqrt{5gl}$,	Tension at highest point C will be zero and body will just complete the circle.
$\sqrt{2gl} < u_A < \sqrt{5gl}$,	Particle will not follow circular motion. Tension in string become zero somewhere between points B and C whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory.

$u_A = \sqrt{2gl}$	Both velocity and tension in the string becomes zero at *B* and particle will oscillate along semi-circular path.
$u_A < \sqrt{2gl}$	velocity of particle becomes zero between *A* and *B* but tension will not be zero and the particle will oscillate about the point *A*.

Note : ❑ *K.E.* of a body moving in horizontal circle is same throughout the path but the *K.E.* of the body moving in vertical circle is different at different places.

❑ If body of mass *m* is tied to a string of length *l* and is projected with a horizontal velocity *u* then :

Height at which the velocity vanishes is $h = \frac{u^2}{2g}$

Height at which the tension vanishes is $h = \frac{u^2 + gl}{3g}$

(3) **Critical condition for vertical looping :** If the tension at *C* is zero, then body will just complete revolution in the vertical circle. This state of body is known as critical state. The speed of body in critical state is called as critical speed.

From the above table 3.3 $T_c = \frac{mu^2}{l} - 5mg = 0$

$\Rightarrow u = \sqrt{5gl}$

It means to complete the vertical circle the body must be projected with minimum velocity of $\sqrt{5gl}$ at the lowest point.

Table 3.5 : Different variables in vertical loop

Quantity	Point *A*	Point *B*	Point *C*	Point *D*	Point *P*
Linear velocity (*v*)	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$	$\sqrt{3gl}$	$\sqrt{gl(3+2\cos\theta)}$
Angular velocity (ω)	$\sqrt{\frac{5g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}(3+2\cos\theta)}$
Tension in String (*T*)	6 *mg*	3 *mg*	0	3 *mg*	$3mg(1+\cos\theta)$
Kinetic Energy (*KE*)	$\frac{5}{2}mgl$	$\frac{3}{2}mgl$	$\frac{1}{2}mgl$	$\frac{3}{2}mgl$	$\frac{mu^2}{l} - 5mg = 0$
Potential Energy (*PE*)	0	*mgl*	2 *mgl*	*mgl*	$mgl(1-\cos\theta)$
Total Energy (*TE*)	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$

(4) **Motion of a block on frictionless hemisphere :** A small block of mass *m* slides down from the top of a frictionless hemisphere of radius *r*. The component of the force of gravity ($mg\cos\theta$) provides required centripetal force but at point *B* it's circular motion ceases and the block lose contact with the surface of the sphere.

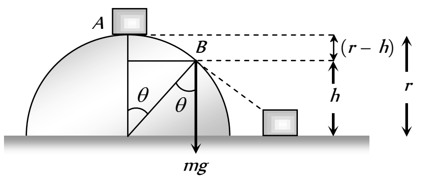


Fig : 3.42

For point *B*, by equating the forces,

$$mg\cos\theta = \frac{mv^2}{r} \quad \text{...(i)}$$

For point *A* and *B*, by law of conservation of energy

Total energy at point *A* = Total energy at point *B*

$K.E._A + P.E._A = K.E._B + P.E._B$

$$0 + mgr = \frac{1}{2}mv^2 + mgh \Rightarrow v = \sqrt{2g(r-h)} \quad \text{...(ii)}$$

and from the given figure $h = r\cos\theta$ ...(iii)

By substituting the value of *v* and *h* from *eq* (ii) and (iii) in *eq* (i)

$$mg\left(\frac{h}{r}\right) = \frac{m}{r}\left(\sqrt{2g(r-h)}\right)^2 \Rightarrow h = 2(r-h) \Rightarrow h = \frac{2}{3}r$$

i.e. the block lose contact at the height of $\frac{2}{3}r$ from the ground.

and angle from the vertical can be given by $\cos\theta = \frac{h}{r} = \frac{2}{3}$

$\therefore \theta = \cos^{-1}\frac{2}{3}$.

Conical Pendulum

This is the example of uniform circular motion in horizontal plane.

A bob of mass *m* attached to a light and in-extensible string rotates in a horizontal circle of radius *r* with constant angular speed ω about the vertical. The string makes angle θ with vertical and appears tracing the surface of a cone. So this arrangement is called conical pendulum.

The force acting on the bob are tension and weight of the bob.

From the figure $T\sin\theta = \frac{mv^2}{r}$ ....(i)

and $T\cos\theta = mg$ ....(ii)

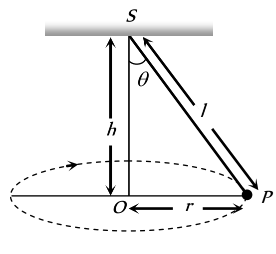


Fig : 3.43

(1) Tension in the string : $T = mg\sqrt{1 + \left(\frac{v^2}{rg}\right)^2}$

$$T = \frac{mg}{\cos\theta} = \frac{mgl}{\sqrt{l^2 - r^2}} \qquad \left[\text{As } \cos\theta = \frac{h}{l} = \frac{\sqrt{l^2 - r^2}}{l}\right]$$

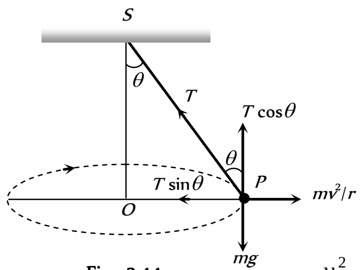


Fig : 3.44

(2) Angle of string from the vertical : $\tan\theta = \frac{v^2}{rg}$

(3) Linear velocity of the bob : $v = \sqrt{gr\tan\theta}$

(4) Angular velocity of the bob :

$$\omega = \sqrt{\frac{g}{r}\tan\theta} = \sqrt{\frac{g}{h}} = \sqrt{\frac{g}{l\cos\theta}}$$

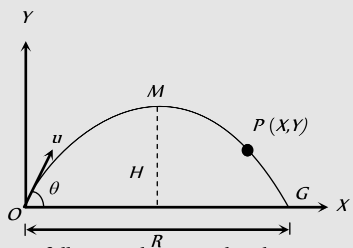
(5) Time period of revolution :

$$T_P = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$$

$$= 2\pi\sqrt{\frac{\sqrt{l^2 - r^2}}{g}} = 2\pi\sqrt{\frac{r}{g\tan\theta}}$$

Tips & Tricks

✍ Consider a projectile of mass m thrown with velocity u making angle θ with the horizontal. It is projected from the point O and returns to the ground at G. Also M is the highest point attained by it. (See figure).



(i) In going from O to M, following changes take place –

(a) Change in velocity $= u\sin\theta$

(b) Change in speed $= u(1 - \cos\theta) = 2u\cos^2(\theta/2)$

(c) Change in momentum $= mu\sin\theta$

(d) Change (loss) in kinetic energy $= 1/2\, mu^2\sin^2\theta$

(e) Change (gain) in potential energy $= 1/2\, mu^2\sin^2\theta$

(f) Change in the direction of motion $= \angle\theta$

(ii) On return to the ground, that is in going from O to G, the following changes take place

(a) Change in speed = zero

(b) Change in velocity = $2u\sin\theta$

(c) Change in momentum = $2mu\sin\theta$

(d) Change in kinetic energy = zero

(e) Change in potential energy = zero

(f) Change in the direction of motion = $\angle 2\theta$

✍ (i) At highest point, the horizontal component of velocity is $v_x = u\cos\theta$ and vertical component of velocity v_y is zero.

(ii) At highest point, linear momentum of a particle

$m\,v_x = mu\cos\theta$.

(iii) Kinetic energy of the particle at the highest point $= \frac{1}{2}mv_x^2$

$$= \frac{1}{2}mu^2\cos^2\theta.$$

✍ At highest point, acceleration due to gravity acting vertically downward makes an angle of 90° with the horizontal component of the velocity of the projectile.

✍ At the highest point, momentum of the projectile thrown at an angle θ with horizontal is $p\cos\theta$ and K.E. = (K.E.) $\cos^2\theta$.

✍ In projectile motion, horizontal component $u\cos\theta$ of velocity u remains constant throughout, whereas vertical component $u\sin\theta$ changes and becomes zero at the highest point.

✍ The trajectory of a projectile is parabolic.

✍ For a projectile, time of flight and maximum height depend on the vertical component of the velocity of projection.

✍ The range of the projectile is maximum for the angle of projection $\theta = 45°$.

✍ The maximum range of the projectile is :

$$R_{max} = \frac{u^2}{g}$$

✍ When the range is maximum, the height attained by the projectile is :

$$H = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

✍ When the range of the projectile is maximum, the time of flight is :

$$T = 2t = \frac{\sqrt{2}u}{g}$$

✍ The height attained by a projectile is maximum, when $\theta = 90°$.

$$H_{max} = \frac{u^2}{2g}$$

It is twice that of height attained, when the range is maximum.

✍ The time of flight of the projectile is also largest for $\theta = 90°$.

$$T_{max} = \frac{2u}{g}$$

✍ The trajectory of the projectile is a symmetric parabola only when g is constant through out the motion and θ is not equal to 0°, 90° or 180°.

✍ If velocity of projection is made n times, the maximum height attained and the range become n times and the time of flight becomes n times the initial value.

✍ If the force acting on a particle is always perpendicular to the velocity of the particle, then the path of the particle is a circle. The centripetal force is always perpendicular to the velocity of the particle.

✍ If circular motion of the object is uniform, the object will possess only centripetal acceleration.

✍ If circular motion of the object is non-uniform, the object will possess both centripetal and transverse acceleration.

✍ When the particle moves along the circular path with constant speed, the angular velocity is also constant. But linear velocity, momentum as well as centripetal acceleration change in direction, although their magnitude remains unchanged.

✍ For circular motion of rigid bodies with uniform speed, the angular speed is same for all particles, but linear speed varies directly as the radius of the circular path described by the particle $(v \propto r)$.

✍ When a body rotates, all its particles describe circular paths about a line, called axis of rotation.

✍ The centre of the circle describe by the different particles of the rotating body lie on the axis of rotation.

✍ Centripetal force $F_c = ma_c,\ m\omega^2 r$ where m = mass of the body.

✍ Centripetal force is always directed towards the centre of the circular path.

✍ When a body rotates with uniform velocity, its different particles have centripetal acceleration directly proportional to the radius $(a_c \propto r)$.

✍ There can be no circular motion without centripetal force.

✍ Centripetal force can be mechanical, electrical or magnetic force.

✍ Planets go round the earth in circular orbits due to the centripetal force provided by gravitational force of the sun.

✍ Gravitational pull of earth provides centripetal force for the orbital motion of the moon and artificial satellites.

✍ Centripetal force cannot change the kinetic energy of the body.

✍ In uniform circular motion the magnitude of the centripetal acceleration remains constant whereas its direction changes continuously but always directed towards the centre.

✍ A pseudo force, that is equal and opposite to the centripetal force is called centrifugal force.

✍ The $\vec{\theta}$, $\vec{\omega}$ and $\vec{\alpha}$ are directed along the axis of the circular path. Their sense of direction is given by the right hand fist rule as follows : 'If we catch axis of rotation in right hand fist such that the fingers point in the direction of rotation, then the outstretched thumb gives the direction of $\vec{\theta}$, $\vec{\omega}$ and $\vec{\alpha}$

✍ $\vec{\theta}$, $\vec{\omega}$ and $\vec{\alpha}$ are called pseudo vectors or axial vectors.

✍ For circular motion we have –

(i) $\vec{r} \perp \vec{v}$ (ii) $\vec{r}$ antiparallel to $\vec{a}_c$

(iii) $\vec{a}_c \perp \vec{v}$ (iv) $\vec{a}_c \perp \vec{a}_t$

(v) $\vec{\theta}$, $\vec{\omega}$, $\vec{\alpha}$ are perpendicular to $\vec{r}, \vec{a}_c, \vec{a}_t, \vec{v}$

(vi) $\vec{r}, \vec{a}_c, \vec{a}_t$ and $\vec{v}$ lie in the same plane

Ordinary Thinking

Objective Questions

Uniform Circular Motion

1. If the body is moving in a circle of radius r with a constant speed v, its angular velocity is [CPMT 1975; RPET 1999]
   (a) v^2/r (b) vr
   (c) v/r (d) r/v

2. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same duration of time t. The ratio of the angular speed of the first to the second car is
   CBSE PMT 1999; UPSEAT 2000]
   (a) $m_1 : m_2$ (b) $r_1 : r_2$
   (c) 1 : 1 (d) $m_1 r_1 : m_2 r_2$

3. A cyclist turns around a curve at 15 miles/hour. If he turns at double the speed, the tendency to overturn is
   [CPMT 1974; AFMC 2003]
   (a) Doubled (b) Quadrupled
   (c) Halved (d) Unchanged

4. A body of mass m is moving in a circle of radius r with a constant speed v. The force on the body is $\frac{mv^2}{r}$ and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle
   (a) $\frac{mv^2}{r} \times \pi r$ (b) Zero
   (c) $\frac{mv^2}{r^2}$ (d) $\frac{\pi r^2}{mv^2}$

5. If a particle moves in a circle describing equal angles in equal times, its velocity vector
   [CPMT 1972, 74; JIPMER 1997]
   (a) Remains constant
   (b) Changes in magnitude
   (c) Changes in direction
   (d) Changes both in magnitude and direction

6. A stone of mass m is tied to a string of length l and rotated in a circle with a constant speed v. If the string is released, the stone flies [NCERT 1977]
   (a) Radially outward
   (b) Radially inward
   (c) Tangentially outward
   (d) With an acceleration $\frac{mv^2}{l}$

7. A body is moving in a circular path with a constant speed. It has
   (a) A constant velocity
   (b) A constant acceleration
   (c) An acceleration of constant magnitude
   (d) An acceleration which varies with time

8. A motor cyclist going round in a circular track at constant speed has
   (a) Constant linear velocity (b) Constant acceleration
   (c) Constant angular velocity (d) Constant force

9. A particle P is moving in a circle of radius 'a' with a uniform speed v. C is the centre of the circle and AB is a diameter. When passing through B the angular velocity of P about A and C are in the ratio [NCERT 1982]
   (a) 1 : 1 (b) 1 : 2
   (c) 2 : 1 (d) 4 : 1

10. A car moving on a horizontal road may be thrown out of the road in taking a turn [NCERT 1983]
    (a) By the gravitational force
    (b) Due to lack of sufficient centripetal force
    (c) Due to rolling frictional force between tyre and road
    (d) Due to the reaction of the ground

11. Two particles of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same speed. The ratio of their centripetal forces is [NCERT 1980; MNR 1995; [NCERT 1984]
    (a) $\frac{r_2}{r_1}$ (b) $\sqrt{\frac{r_2}{r_1}}$
    (c) $\left(\frac{r_1}{r_2}\right)^2$ (d) $\left(\frac{r_2}{r_1}\right)^2$

12. A particle moves with constant angular velocity in a circle. During the motion its
    (a) Energy is conserved
    (b) Momentum is conserved
    (c) Energy and momentum both are conserved
    (d) None of the above is conserved

13. A stone tied to a string is rotated in a circle. If the string is cut, the stone flies away from the circle because [NCERT 1977; RPET 1999]
    (a) A centrifugal force acts on the stone
    (b) A centripetal force acts on the stone
    (c) Of its inertia
    (d) Reaction of the centripetal force

14. A body is revolving with a constant speed along a circle. If its direction of motion is reversed but the speed remains the same, then which of the following statement is true
    (a) The centripetal force will not suffer any change in magnitude
    (b) The centripetal force will have its direction reversed
    (c) The centripetal force will not suffer any change in direction
    (d) The centripetal force would be doubled

15. When a body moves with a constant speed along a circle
    [CBSE PMT 1994; Orissa PMT 2004]
    (a) No work is done on it
    (b) No acceleration is produced in the body
    (c) No force acts on the body
    (d) Its velocity remains constant

16. A body of mass m moves in a circular path with uniform angular velocity. The motion of the body has constant
    [MP PET 2003]
    (a) Acceleration [CPMT 1972] (b) Velocity
    (c) Momentum (d) Kinetic energy

17. On a railway curve, the outside rail is laid higher than the inside one so that resultant force exerted on the wheels of the rail car by the tops of the rails will [NCERT 1975]
    (a) Have a horizontal inward component

(b) Be vertical
(c) Equilibriate the centripetal force
(d) Be decreased

18. If the overbridge is concave instead of being convex, the thrust on the road at the lowest position will be

(a) $mg+\frac{mv^2}{r}$ (b) $mg-\frac{mv^2}{r}$
(c) $\frac{m^2v^2g}{r}$ (d) $\frac{v^2g}{r}$

19. A cyclist taking turn bends inwards while a car passenger taking same turn is thrown outwards. The reason is [NCERT 1972; CPMT 1974]

(a) Car is heavier than cycle
(b) Car has four wheels while cycle has only two
(c) Difference in the speed of the two
(d) Cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force

20. A car sometimes overturns while taking a turn. When it overturns, it is [AFMC 1988; MP PMT 2003]

(a) The inner wheel which leaves the ground first
(b) The outer wheel which leaves the ground first
(c) Both the wheels leave the ground simultaneously
(d) Either wheel leaves the ground first

21. A tachometer is a device to measure [DPMT 1999]

(a) Gravitational pull (b) Speed of rotation
(c) Surface tension (d) Tension in a spring

22. Two bodies of mass 10 kg and 5 kg moving in concentric orbits of radii R and r such that their periods are the same. Then the ratio between their centripetal acceleration is [CBSE PMT 2001]

(a) R/r (b) r/R
(c) R^2/r^2 (d) r^2/R^2

23. The ratio of angular speeds of minute hand and hour hand of a watch is [MH CET 2002]

(a) 1 : 12 (b) 6 : 1
(c) 12 : 1 (d) 1 : 6

24. A car travels north with a uniform velocity. It goes over a piece of mud which sticks to the tyre. The particles of the mud, as it leaves the ground are thrown

(a) Vertically upwards (b) Vertically inwards
(c) Towards north (d) Towards south

25. An aircraft executes a horizontal loop with a speed of 150 *m/s* with its, wings banked at an angle of 12^o. The radius of the loop is $(g=10\ m/s^2)$ [Pb. PET 2001]

(a) 10.6 *km* (b) 9.6 *km*
(c) 7.4 *km* (d) 5.8 *km*

26. A particle is moving in a horizontal circle with constant speed. It has constant [MP PMT 1987; AFMC 1993; CPMT 1997; MP PET 2000]

(a) Velocity (b) Acceleration
(c) Kinetic energy (d) Displacement

27. A motor cyclist moving with a velocity of 72 *km/hour* on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 *meters*. The acceleration due to gravity is 10 *m/sec*. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than

(a) $\theta=\tan^{-1}6$ (b) $\theta=\tan^{-1}2$
(c) $\theta=\tan^{-1}25.92$ (d) $\theta=\tan^{-1}4$

28. A train is moving towards north. At one place it turns towards north-east, here we observe that [AIIMS 1980]

(a) The radius of curvature of outer rail will be greater than that of the inner rail
(b) The radius of the inner rail will be greater than that of the outer rail
(c) The radius of curvature of one of the rails will be greater
(d) The radius of curvature of the outer and inner rails will be the same

29. The angular speed of a fly wheel making 120 *revolutions/minute* is [CBSE P

(a) $2\pi\ rad/s$ (b) $4\pi^2\ rad/s$
(c) $\pi\ rad/s$ (d) $4\pi\ rad/s$

30. A particle is moving on a circular path with constant speed, then its acceleration will be [RPET 2003]

(a) Zero
(b) External radial acceleration
(c) Internal radial acceleration
(d) Constant acceleration

31. A car is moving on a circular path and takes a turn. If R_1 and R_2 be the reactions on the inner and outer wheels respectively, then

(a) $R_1=R_2$ (b) $R_1<R_2$
(c) $R_1>R_2$ (d) $R_1\geq R_2$

32. A mass of 100 *gm* is tied to one end of a string 2 *m* long. The body is revolving in a horizontal circle making a maximum of 200 revolutions per min. The other end of the string is fixed at the centre of the circle of revolution. The maximum tension that the string can bear is (approximately) [MP PET 1993]

(a) 8.76 *N* (b) 8.94 *N*
(c) 89.42 *N* (d) 87.64 *N*

33. A road is 10 *m* wide. Its radius of curvature is 50 *m*. The outer edge is above the lower edge by a distance of 1.5 *m*. This road is most suited for the velocity

(a) 2.5 *m/sec* (b) 4.5 *m/sec*
(c) 6.5 *m/sec* (d) 8.5 *m/sec*

34. Certain neutron stars are believed to be rotating at about $1\,rev/\sec$. If such a star has a radius of 20 *km*, the acceleration of an object on the equator of the star will be [NCERT 1982]

(a) $20\times10^8\,m/\sec^2$ (b) $8\times10^5\,m/\sec^2$
(c) $120\times10^5\,m/\sec^2$ (d) $4\times10^8\,m/\sec^2$

35. A particle revolves round a circular path. The acceleration of the particle is [MNR 1986; UPSEAT 1999]

(a) Along the circumference of the circle
(b) Along the tangent
(c) Along the radius
(d) Zero

36. The length of second's hand in a watch is 1 *cm*. The change in velocity of its tip in 15 seconds is [MP PMT 1987, 2003]

(a) Zero (b) $\frac{\pi}{30\sqrt{2}} cm / \sec$

(c) $\frac{\pi}{30} cm / \sec$ (d) $\frac{\pi\sqrt{2}}{30} cm / \sec$

37. A particle moves in a circle of radius 25 *cm* at two revolutions per second. The acceleration of the particle in m / s^2 is [MNR 1991; UPSEAT 2000; DPMT 1999; RPET 2003; Pb. PET 2004]

(a) π^2 (b) $8\pi^2$

(c) $4\pi^2$ (d) $2\pi^2$

38. An electric fan has blades of length 30 *cm* as measured from the axis of rotation. If the fan is rotating at 1200 *r.p.m.* The acceleration of a point on the tip of the blade is about [CBSE PMT 1990]

(a) 1600 $m / \sec^2$ (b) 4740 $m / \sec^2$

(c) 2370 $m / \sec^2$ (d) 5055 $m / \sec^2$

39. The force required to keep a body in uniform circular motion is [EAMCET 1982; AFMC 2003]

(a) Centripetal force (b) Centrifugal force

(c) Resistance (d) None of the above

40. Cream gets separated out of milk when it is churned, it is due to [EAMCET 1981]

(a) Gravitational force (b) Centripetal force

(c) Centrifugal force (d) Frictional force

41. A particle of mass m is executing uniform circular motion on a path of radius r. If p is the magnitude of its linear momentum. The radial force acting on the particle is [MP PET 1994]

(a) pmr (b) $\frac{rm}{p}$

(c) $\frac{mp^2}{r}$ (d) $\frac{p^2}{rm}$

42. A particle moves in a circular orbit under the action of a central attractive force inversely proportional to the distance 'r'. The speed of the particle is [CBSE PMT 1995]

(a) Proportional to r^2 (b) Independent of r

(c) Proportional to r (d) Proportional to $1/r$

43. Two masses M and m are attached to a vertical axis by weightless threads of combined length l. They are set in rotational motion in a horizontal plane about this axis with constant angular velocity ω. If the tensions in the threads are the same during motion, the distance of M from the axis is [MP PET 1995]

(a) $\frac{Ml}{M+m}$ (b) $\frac{ml}{M+m}$

(c) $\frac{M+m}{M} l$ (d) $\frac{M+m}{m} l$

44. A boy on a cycle pedals around a circle of 20 *metres* radius at a speed of $20 \, metres / \sec$. The combined mass of the boy and the cycle is 90 *kg*. The angle that the cycle makes with the vertical so that it may not fall is $(g = 9.8 \, m / \sec^2)$ [MP PMT 1995]

(a) 60.25^o (b) 63.90^o

(c) 26.12^o (d) 30.00^o

45. The average acceleration vector for a particle having a uniform circular motion is [Kurukshetra CEE 1996]

(a) A constant vector of magnitude $\frac{v^2}{r}$

(b) A vector of magnitude $\frac{v^2}{r}$ directed normal to the plane of the given uniform circular motion

(c) Equal to the instantaneous acceleration vector at the start of the motion

(d) A null vector

46. Radius of the curved road on national highway is R. Width of the road is b. The outer edge of the road is raised by h with respect to inner edge so that a car with velocity v can pass safe over it. The value of h is [MP PMT 1996]

(a) $\frac{v^2 b}{Rg}$ (b) $\frac{v}{Rgb}$

(c) $\frac{v^2 R}{g}$ (d) $\frac{v^2 b}{R}$

47. When a particle moves in a uniform circular motion. It has

(a) Radial velocity and radial acceleration

(b) Tangential velocity and radial acceleration

(c) Tangential velocity and tangential acceleration

(d) Radial velocity and tangential acceleration

48. A motorcycle is going on an overbridge of radius R. The driver maintains a constant speed. As the motorcycle is ascending on the overbridge, the normal force on it [MP PET 1997]

(a) Increases (b) Decreases

(c) Remains the same (d) Fluctuates

49. A mass of 2 *kg is* whirled in a horizontal circle by means of a string at an initial speed of 5 *revolutions per minute*. Keeping the radius constant the tension in the string is doubled. The new speed is nearly [MP PMT/PET 1998; JIPMER 2000]

(a) 14 *rpm* (b) 10 *rpm*

(c) 2.25 *rpm* (d) 7 *rpm*

50. The magnitude of the centripetal force acting on a body of mass m executing uniform motion in a circle of radius r with speed v is [AFMC 1998

(a) mvr (b) mv^2/r

(c) v/r^2m (d) v/rm

51. A string breaks if its tension exceeds 10 *newtons*. A stone of mass 250 *gm* tied to this string of length 10 *cm* is rotated in a horizontal circle. The maximum angular velocity of rotation can be

(a) 20 *rad/s* (b) 40 *rad/s*

(c) 100 *rad/s* (d) 200 *rad/s*

52. A 500 *kg* car takes a round turn of radius 50 *m* with a velocity of 36 *km/hr*. The centripetal force is [KCET 2001; CBSE PMT 1999;

JIPMER 2001, 02]

(a) 250 *N* (b) 750 *N*

(c) 1000 *N* (d) 1200 *N*

53. A ball of mass 0.25 *kg* attached to the end of a string of length 1.96 *m* is moving in a horizontal circle. The string will break if the tension is more than 25 *N*. What is the maximum speed with which the ball can be moved [CBSE PMT 1998]

(a) 14 *m/s* (b) 3 *m/s*

(c) 3.92 *m/s* (d) 5 *m/s*

54. A body of mass 5 *kg* is moving in a circle of radius $1\,m$ with an angular velocity of 2 *radian/sec*. The centripetal force is [AIIMS 1998]

(a) 10 *N* (b) 20 *N*

(c) 30 *N* (d) 40 *N*

55. If a particle of mass m is moving in a horizontal circle of radius r with a centripetal force $(-k/r^2)$, the total energy is [EAMCET (Med.) 1995; AMU (Engg.) 2001]

(a) $-\frac{k}{2r}$ (b) $-\frac{k}{r}$

(c) $-\frac{2k}{r}$ (d) $-\frac{4k}{r}$

56. A stone of mass of 16 *kg* is attached to a string 144 *m* long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16 *Newton*. The maximum velocity of revolution that can be given to the stone without breaking it, will be

(a) 20 ms^{-1} (b) 16 ms^{-1}

(c) 14 ms^{-1} (d) 12 ms^{-1}

57. A circular road of radius 1000 *m* has banking angle 45^o. The maximum safe speed of a car having mass 2000 *kg* will be, if the coefficient of friction between tyre and road is 0.5 [RPET 1997]

(a) 172 *m/s* (b) 124 *m/s*

(c) 99 *m/s* (d) 86 *m/s*

58. The second's hand of a watch has length 6 *cm*. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be [RPET 1997]

(a) 6.28 and 0 *mm/s* (b) 8.88 and 4.44 *mm/s*

(c) 8.88 and 6.28 *mm/s* (d) 6.28 and 8.88 *mm/s*

59. A sphere of mass m is tied to end of a string of length l and rotated through the other end along a horizontal circular path with speed v. The work done in full horizontal circle is [CPMT 1993; JIPMER 2000]

(a) 0 (b) $\left(\frac{mv^2}{l}\right).2\pi l$

(c) $mg\,.\,2\pi l$ (d) $\left(\frac{mv^2}{l}\right).(l)$

60. A body is whirled in a horizontal circle of radius 20 *cm*. It has angular velocity of 10 *rad/s*. What is its linear velocity at any point on circular path [CBSE PMT 1996]

(a) 10 *m/s* (b) 2 *m/s*

(c) 20 *m/s* (d) $\sqrt{2}$ *m/s*

61. Find the maximum velocity for skidding for a car moved on a circular track of radius 100 *m*. The coefficient of friction between the road and tyre is 0.2 [CPMT 1996; Pb. PMT 2001]

(a) 0.14 *m/s* (b) 140 *m/s*

(c) 1.4 *km/s* (d) 14 *m/s*

62. A car when passes through a convex bridge exerts a force on it which is equal to [AFMC 1997]

(a) $Mg+\frac{Mv^2}{r}$ (b) $\frac{Mv^2}{r}$

(c) Mg (d) None of these

63. The angular speed of seconds needle in a mechanical watch is [RPMT 1999; CPMT 1997; MH CET 2000, 01; BHU 2000]

(a) $\frac{\pi}{30}$ *rad/s* (b) 2π *rad/s*

(c) π *rad/s* (d) $\frac{60}{\pi}$ *rad/s*

64. The angular velocity of a particle rotating in a circular orbit 100 times per minute is [SCRA 1998; DPMT 2000]

(a) 1.66 *rad/s* (b) 10.47 *rad/s*

(c) 10.47 *deg/s* (d) 60 *deg/s*

65. A body of mass 100 *g* is rotating in a circular path of radius *r* with constant velocity. The work done in one complete revolution is [SCRA 1994]

(a) 100*r J* (b) $(r/100)J$

(c) $(100/r)J$ (d) Zero

66. A particle comes round a circle of radius 1 *m* once. The time taken by it is 10 *sec*. The average velocity of motion is [JIPMER 1999]

(a) $0.2\ \pi m/s$ (b) $2\ \pi m/s$

(c) $2\ m/s$ (d) Zero

67. An unbanked curve has a radius of $60\,m$. The maximum speed at which a car can make a turn if the coefficient of static friction is 0.75, is [JIPMER 1999]

(a) 2.1 m/s (b) 14 m/s

(c) 21 m/s (d) 7 m/s

68. A wheel completes 2000 revolutions to cover the 9.5 *km*. distance. then the diameter of the wheel is [RPMT 1999]

(a) 1.5 *m* (b) 1.5 *cm*

(c) 7.5 *cm* (d) 7.5 *m*

69. A cycle wheel of radius 0.4 *m* completes one revolution in one *second* then the acceleration of a point on the cycle wheel will be

(a) 0.8 *m/s* (b) 0.4 *m/s*

(c) $1.6\pi^2\,m/s^2$ (d) $0.4\pi^2\,m/s^2$

70. The centripetal acceleration is given by [RPET 1999]

(a) v/r (b) vr

(c) vr (d) v/r

71. A cylindrical vessel partially filled with water is rotated about its vertical central axis. It's surface will [RPET 2000]

(a) Rise equally (b) Rise from the sides

(c) Rise from the middle (d) Lowered equally

72. If a particle covers half the circle of radius R with constant speed then [RPMT 2000]
(a) Momentum change is *mvr*
(b) Change in *K.E.* is 1/2 *mv*
(c) Change in *K.E.* is *mv*
(d) Change in *K.E.* is zero

73. An aeroplane is flying with a uniform speed of 100 *m/s* along a circular path of radius 100 *m*. the angular speed of the aeroplane will be [KCET 2000]
(a) 1 *rad/sec* (b) 2 *rad/sec*
(c) 3 *rad/sec* (d) 4 *rad/sec*

74. A body moves with constant angular velocity on a circle. Magnitude of angular acceleration [RPMT 2000]
(a) $r\omega$ (b) Constant
(c) Zero (d) None of the above

75. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$ [Pb. PMT 2000]
(a) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (b) $-18\hat{i} - 13\hat{j} + 2\hat{k}$
(c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$

76. A stone is tied to one end of a string 50 *cm* long is whirled in a horizontal circle with a constant speed. If the stone makes 10 revolutions in 20 *s*, what is the magnitude of acceleration of the stone [Pb. PMT 2000]
(a) 493 *cm/s* (b) 720 *cm/s*
(c) 860 *cm/s* (d) 990 *cm/s*

77. A 100 *kg* car is moving with a maximum velocity of 9 *m/s* across a circular track of radius 30 *m*. The maximum force of friction between the road and the car is [Pb. PMT 2000]
(a) 1000 *N* (b) 706 *N*
(c) 270 *N* (d) 200 *N*

78. The maximum speed of a car on a road–turn of radius 30 *m*, if the coefficient of friction between the tyres and the road is 0.4, will be
(a) 10.84 *m/sec* (b) 9.84 *m/sec*
(c) 8.84 *m/sec* (d) 6.84 *m/sec*

79. The angular velocity of a wheel is 70 *rad/sec*. If the radius of the wheel is 0.5 *m*, then linear velocity of the wheel is [MH CET 2000]
(a) 70 *m/s* (b) 35 *m/s*
(c) 30 *m/s* (d) 20 *m/s*

80. A cyclist goes round a circular path of circumference 34.3 *m* in $\sqrt{22}$ *sec*. the angle made by him, with the vertical, will be
(a) 45 (b) 40
(c) 42 (d) 48

81. A particle of mass M is moving in a horizontal circle of radius R with uniform speed V. When it moves from one point to a diametrically opposite point, its [CBSE PMT 1992]
(a) Kinetic energy changes by $MV^2/4$
(b) Momentum does not change
(c) Momentum changes by $2MV$
(d) Kinetic energy changes by MV^2

82. A ball of mass 0.1 *Kg*. is whirled in a horizontal circle of radius 1 *m*. by means of a string at an initial speed of 10 *R.P.M.* Keeping the radius constant, the tension in the string is reduced to one quarter of its initial value. The new speed is
(a) 5 *r.p.m.* (b) 10 *r.p.m.*
(c) 20 *r.p.m.* (d) 14 *r.p.m.*

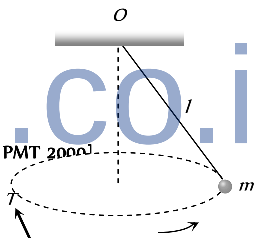
83. A cyclist riding the bicycle at a speed of $14\sqrt{3}$ *ms* takes a turn around a circular road of radius $20\sqrt{3}$ *m* without skidding. Given *g* = 9.8 *ms*, what is his inclination to the vertical
(a) 30 (b) 90
(c) 45 (d) 60

84. If a cycle wheel of radius 4 *m* completes one revolution in two seconds. Then acceleration of a point on the cycle wheel will be
(a) $\pi^2 m/s^2$ (b) $2\pi^2 m/s^2$
(c) $4\pi^2 m/s^2$ (d) $8\pi m/s^2$

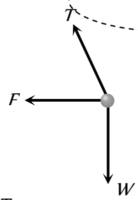
85. A bob of mass 10 *kg* is attached to wire 0.3 *m* long. Its breaking stress is 4.8 × 10 *N/m*. The area of cross section of the wire is 10 *m*. The maximum angular velocity with which it can be rotated in a horizontal circle [Pb. PMT 2001]
(a) 8 *rad/sec* (b) 4 *rad/sec*
(c) 2 *rad/sec* (d) 1 *rad/sec*

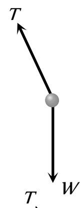
86. In uniform circular motion, the velocity vector and acceleration vector are [DCE 2000, 01, 03]
(a) Perpendicular to each other
(b) Same direction
(c) Opposite direction
(d) Not related to each other

87. A point mass *m* is suspended from a light thread of length *l*, fixed at *O*, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are [AMU (Med.) 2001]

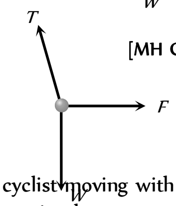


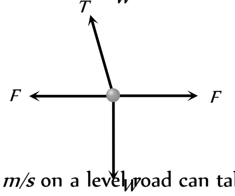
[CBSE PMT 2000]

(a)


(b)


[MH CET 2000]

(c)


(d)


88. If a cyclist moving with a speed of 4.9 *m/s* on a level road can take a sharp circular turn of radius 4 *m*, then coefficient of friction between the cycle tyres and road is [AIIMS 1999; AFMC 2001]
(a) 0.41 (b) 0.51
(c) 0.61 (d) 0.71

89. A car moves on a circular road. It describes equal angles about the centre in equal intervals of time. Which of the following statement about the velocity of the car is true [BHU 2001]
[MP PMT 2001]
(a) Magnitude of velocity is not constant
(b) Both magnitude and direction of velocity change
(c) Velocity is directed towards the centre of the circle
(d) Magnitude of velocity is constant but direction changes

90. A scooter is going round a circular road of radius 100 *m* at a speed of 10 *m/s*. The angular speed of the scooter will be [Pb. PMT 2002]

(a) 0.01 *rad/s* (b) 0.1 *rad/s*

(c) 1 *rad/s* (d) 10 *rad/s*

91. A particle of mass *M* moves with constant speed along a circular path of radius *r* under the action of a force *F*. Its speed is

(a) $\sqrt{\frac{rF}{m}}$ (b) $\sqrt{\frac{F}{r}}$

(c) $\sqrt{Fmr}$ (d) $\sqrt{\frac{F}{mr}}$

92. In an atom for the electron to revolve around the nucleus, the necessary centripetal force is obtained from the following force exerted by the nucleus on the electron [MP PET 2002]

(a) Nuclear force (b) Gravitational force

(c) Magnetic force (d) Electrostatic force

93. A particle moves with constant speed *v* along a circular path of radius *r* and completes the circle in time *T*. The acceleration of the particle is [Orissa JEE 2002]

(a) $2\pi v/T$ (b) $2\pi r/T$

(c) $2\pi r^2/T$ (d) $2\pi v^2/T$

94. The maximum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 *m* and coefficient of friction 0.6 to avoid skidding is [AIEEE 2002]

(a) 60 (b) 30

(c) 15 (d) 25

95. A car is moving with high velocity when it has a turn. A force acts on it outwardly because of [AFMC 2002]

(a) Centripetal force (b) Centrifugal force

(c) Gravitational force (d) All the above

96. A motor cycle driver doubles its velocity when he is having a turn. The force exerted outwardly will be [AFMC 2002]

(a) Double (b) Half

(c) 4 times (d) $\frac{1}{4}$ times

97. The coefficient of friction between the tyres and the road is 0.25. The maximum speed with which a car can be driven round a curve of radius 40 *m* without skidding is (assume $g = 10\ ms^{-2}$)

(a) 40 ms^{-1} (b) 20 ms^{-1}

(c) 15 ms^{-1} (d) 10 ms^{-1}

98. An athlete completes one round of a circular track of radius 10 *m* in 40 *sec*. The distance covered by him in 2 *min* 20 *sec* is

(a) 70 *m* (b) 140 *m*

(c) 110 *m* (d) 220 *m*

99. A proton of mass 1.6×10^{-27} *kg* goes round in a circular orbit of radius 0.10 *m* under a centripetal force of 4×10^{-13} *N*. then the frequency of revolution of the proton is about [Kerala (Med.) 2002]

(a) 0.08×10^{8} *cycles per sec*

(b) 4×10^{8} *cycles per sec*

(c) 8×10^{8} *cycles per sec*

(d) 12×10^{8} *cycles per sec*

100. A particle is moving in a circle with uniform speed *v*. In moving from a point to another diametrically opposite point [Orissa JEE 2003]

(a) The momentum changes by *mv*

(b) The momentum changes by 2*mv*

(c) The kinetic energy changes by (1/2)*mv*

(d) The kinetic energy changes by *mv*

101. In uniform circular motion [MP PMT 2002] [MP PMT 1994]

(a) Both the angular velocity and the angular momentum vary

(b) The angular velocity varies but the angular momentum remains constant

(c) Both the angular velocity and the angular momentum stay constant

(d) The angular momentum varies but the angular velocity remains constant

102. When a body moves in a circular path, no work is done by the force since, [KCET 2004]

(a) There is no displacement

(b) There is no net force

(c) Force and displacement are perpendicular to each other

(d) The force is always away from the centre

103. Which of the following statements is false for a particle moving in a circle with a constant angular speed [AIEEE 2004]

(a) The velocity vector is tangent to the circle

(b) The acceleration vector is tangent to the circle

(c) The acceleration vector points to the centre of the circle

(d) The velocity and acceleration vectors are perpendicular to each other

104. If a_r and a_t represent radial and tangential accelerations, the motion of a particle will be uniformly circular if [CPMT 2004]

(a) $a_r = 0$ and $a_t = 0$ (b) $a_r = 0$ but $a_t \neq 0$

(c) $a_r \neq 0$ but $a_t = 0$ (d) $a_r \neq 0$ and $a_t \neq 0$

105. A person with his hands in his pockets is skating on ice at the velocity of 10 *m/s* and describes a circle of radius 50 *m*. What is his inclination with vertical [Pb. PET 2000]

(a) $\tan^{-1}\left(\frac{1}{10}\right)$ [Kerala (Med.) 2002] (b) $\tan^{-1}\left(\frac{3}{5}\right)$

(c) $\tan^{-1}(1)$ (d) $\tan^{-1}\left(\frac{1}{5}\right)$

106. If the radius of curvature of the path of two particles of same masses are in the ratio 1 : 2, then in order to have constant centripetal force, their velocity, should be in the ratio of [Kerala (Med.) 2002] [Pb. PET 2000]

(a) 1 : 4 (b) 4 : 1

(c) $\sqrt{2} : 1$ (d) $1 : \sqrt{2}$

107. An object is moving in a circle of radius 100 *m* with a constant speed of 31.4 *m/s*. What is its average speed for one complete revolution [DCE 2004]

(a) Zero (b) 31.4 *m/s*

(c) 3.14 *m/s* (d) $\sqrt{2} \times 31.4\ m/s$

108. A body of mass 1 *kg* tied to one end of string is revolved in a horizontal circle of radius 0.1 *m* with a speed of 3 *revolution/sec*, assuming the effect of gravity is negligible, then linear velocity, acceleration and tension in the string will be

(a) $1.88\ m/s,\ 35.5\ m/s^2,\ 35.5\ N$

(b) $2.88\ m/s,\ 45.5\ m/s^2,\ 45.5\ N$

(c) $3.88\ m/s,\ 55.5\ m/s^2,\ 55.5\ N$

(d) None of these

109. The acceleration of a train travelling with speed of 400 *m/s* as it goes round a curve of radius 160 *m*, is [Pb. PET 2003]

(a) $1\ km/s^2$ (b) $100\ m/s^2$

(c) $10\ m/s^2$ (d) $1\ m/s^2$

110. A car of mass 800 *kg* moves on a circular track of radius 40 *m*. If the coefficient of friction is 0.5, then maximum velocity with which the car can move is [MH CET 2004]

(a) 7 *m/s* (b) 14 *m/s*

(c) 8 *m/s* (d) 12 *m/s*

111. A 500 *kg* crane takes a turn of radius 50 *m* with velocity of 36 *km/hr*. The centripetal force is [Pb. PMT 2003]

(a) 1200 *N* (b) 1000 *N*

(c) 750 *N* (d) 250 *N*

112. Two bodies of equal masses revolve in circular orbits of radii R_1 and R_2 with the same period. Their centripetal forces are in the ratio [Kerala PMT 2004]

(a) $\left(\frac{R_2}{R_1}\right)^2$ (b) $\frac{R_1}{R_2}$

(c) $\left(\frac{R_1}{R_2}\right)^2$ (d) $\sqrt{R_1 R_2}$

113. In case of uniform circular motion which of the following physical quantity do not remain constant [Kerala PMT 2004]

(a) Speed (b) Momentum

(c) Kinetic energy (d) Mass

114. What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and half the angular velocity ω

(a) The centripetal acceleration remains unchanged

(b) The centripetal acceleration is halved

(c) The centripetal acceleration is doubled

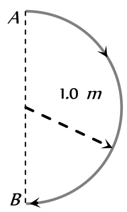
(d) The centripetal acceleration is quadrupled

115. A mass is supported on a frictionless horizontal surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω_0. If the length of the string and angular velocity are doubled, the tension in the string which was initially T_0 is now

(a) T_0 (b) $T_0/2$

(c) $4T_0$ (d) $8T_0$

116. In 1.0 *s*, a particle goes from point *A* to point *B*, moving in a semicircle of radius 1.0 *m* (see figure). The magnitude of the average velocity is [IIT-JEE 1999]

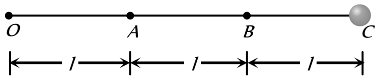


[DPMT 2003]

(a) $3.14\,m/s$ (b) $2.0\ m/s$

(c) $1.0\ m/s$ (d) Zero

117. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is *v*, then the ratio of tensions in the three sections of the string is



(a) 3 : 5 : 7 (b) 3 : 4 : 5

(c) 7 : 11 : 6 (d) 3 : 5 : 6

118. A particle is moving in a circle of radius *R* with constant speed *v*, if radius is double then its centripetal force to keep the same speed should be [BCECE 2005]

(a) Doubled

(b) Halved

(c) Quadrupled

(d) Unchanged

119. A stone ties to the end of a string $1m$ long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 seconds, what is the magnitude and direction of acceleration of the stone [CBSE PMT 2005]

(a) $\frac{\pi^2}{4} ms^{-2}$ and direction along the radius towards the centre

(b) $\pi^2 ms^{-2}$ and direction along the radius away from the centre

(c) $\pi^2 ms^{-2}$ and direction along the radius towards the centre

(d) $\pi^2 ms^{-2}$ and direction along the tangent to the circle

120. A particle describes a horizontal circle in a conical funnel whose inner surface is smooth with speed of 0.5 *m/s*. What is the height of the plane of circle from vertex of the funnel ? [J&K CET 2005]

(a) 0.25 *cm* (b) 2 *cm*

(c) 4 *cm* (d) 2.5 *cm*

121. What is the angular velocity of earth [Orissa JEE 2005]

(a) $\frac{2\pi}{86400} rad/\sec$ (b) $\frac{2\pi}{3600} rad/\sec$

(c) $\frac{2\pi}{24} rad/\sec$ [AIIMS 1985] (d) $\frac{2\pi}{6400} rad/\sec$

122. If the length of the second's hand in a stop clock is 3 *cm* the angular velocity and linear velocity of the tip is [Kerala PET 2005]

(a) 0.2047 *rad/sec.*, 0.0314 *m/sec*

(b) 0.2547 *rad/sec.*, 0.314 *m/sec*

(c) 0.1472 *rad/sec.*, 0.06314 *m/sec*

(d) 0.1047 *rad/sec.*, 0.00314 *m/sec*

Non-uniform Circular Motion

1. In a circus stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be

[CPMT 1979; JIPMER 1997; RPET 1999]

(a) $\sqrt{2gR}$ (b) $2gR$

(c) $\sqrt{3gR}$ (d) $\sqrt{gR}$

2. A block of mass m at the end of a string is whirled round in a vertical circle of radius R. The critical speed of the block at the top of its swing below which the string would slacken before the block reaches the top is [DCE 1999, 2001]

(a) Rg (b) $(Rg)^2$

(c) R/g (d) $\sqrt{Rg}$

3. A sphere is suspended by a thread of length l. What minimum horizontal velocity has to be imparted the ball for it to reach the height of the suspension

[ISM Dhanbad 1994]

(a) gl (b) $2gl$

(c) $\sqrt{gl}$ (d) $\sqrt{2gl}$

4. A bottle of sodawater is grasped by the neck and swing briskly in a vertical circle. Near which portion of the bottle do the bubbles collect

(a) Near the bottom

(b) In the middle of the bottle

(c) Near the neck

(d) Uniformly distributed in the bottle

5. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill, when the bucket is at the highest position (Take $g = 10 m/\sec^2$)

(a) 4 *m/sec* (b) 6.25 *m/sec*

(c) 16 *m/sec* (d) None of the above

6. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 *sec*, it rotates through an angle θ_1. In the next 2 *sec*, it rotates through an additional angle θ_2. The ratio of θ_2/θ_1 is

[AIIMS 1985]

(a) 1 (b) 2

(c) 3 (d) 5

7. A 1 *kg* stone at the end of 1 *m* long string is whirled in a vertical circle at constant speed of 4 *m/sec*. The tension in the string is 6 *N*, when the stone is at (g = 10 *m/sec*)

[AIIMS 1982]

(a) Top of the circle (b) Bottom of the circle

(c) Half way down (d) None of the above

8. A cane filled with water is revolved in a vertical circle of radius 4 *meter* and the water just does not fall down. The time period of revolution will be [CPMT 1985;

RPET 1995; UPSEAT 2002; MH CET 2002]

(a) 1 *sec* (b) 10 *sec*

(c) 8 *sec* (d) 4 *sec*

9. A 2 *kg* stone at the end of a string 1 *m* long is whirled in a vertical circle at a constant speed. The speed of the stone is 4 *m/sec*. The tension in the string will be 52 *N*, when the stone is

(a) At the top of the circle

(b) At the bottom of the circle

(c) Halfway down

(d) None of the above

10. A body slides down a frictionless track which ends in a circular loop of diameter D, then the minimum height h of the body in term of D so that it may just complete the loop, is

(a) $h = \frac{5D}{2}$ (b) $h = \frac{5D}{4}$

(c) $h = \frac{3D}{4}$ (d) $h = \frac{D}{4}$

11. A car is moving with speed 30 $m/\sec$ on a circular path of radius 500 *m*. Its speed is increasing at the rate of $2m/\sec^2$, What is the acceleration of the car

[MP PMT 2003; Roorkee 1982; RPET 1996; MH CET 2002]

(a) $2m/\sec^2$ (b) $2.7m/\sec^2$

(c) $1.8m/\sec^2$ (d) $9.8\,m/\sec^2$

12. The string of pendulum of length l is displaced through 90^o from the vertical and released. Then the minimum strength of the string in order to withstand the tension, as the pendulum passes through the mean position is

[MP PMT 1986]

(a) mg (b) $3mg$

[AIIMS 1987]

(c) $5\,mg$ (d) $6\,mg$

13. A weightless thread can support tension upto 30 *N*. A stone of mass 0.5 *kg* is tied to it and is revolved in a circular path of radius 2 *m* in a vertical plane. If $g = 10m/s^2$, then the maximum angular velocity of the stone will be

[MP PMT 1994]

(a) $5\ rad/s$ (b) $\sqrt{30}\ rad/s$

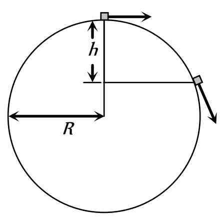
(c) $\sqrt{60}\ rad/s$ (d) $10\,rad/s$

14. A particle originally at rest at the highest point of a smooth vertical circle is slightly displaced. It will leave the circle at a vertical distance h below the highest point such that

(a) $h = R$

(b) $h = \frac{R}{3}$

(c) $h = \frac{R}{2}$



(d) $h = \frac{2R}{3}$

15. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break

[MP PET 1997]

(a) When the mass is at the highest point of the circle

(b) When the mass is at the lowest point of the circle

(c) When the wire is horizontal

(d) At an angle of $\cos^{-1}(1/3)$ from the upward vertical

16. A weightless thread can bear tension upto 3.7 *kg wt*. A stone of mass 500 *gms* is tied to it and revolved in a circular path of radius 4 *m* in a vertical plane. If $g = 10\, ms^{-2}$, then the maximum angular velocity of the stone will be

[MP PMT/PET 1998]

(a) 4 *radians/sec* (b) 16 *radians/sec*

(c) $\sqrt{21}$ *radians/sec* (d) 2 *radians/sec*

17. The maximum velocity at the lowest point, so that the string just slack at the highest point in a vertical circle of radius l

[CPMT 1999; MH CET 2004]

(a) $\sqrt{gl}$ (b) $\sqrt{3gl}$

(c) $\sqrt{5gl}$ (d) $\sqrt{7gl}$

18. If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2 *sec* from its start is [AIIMS 1998]

(a) 8 *rad/sec* (b) 12 *rad/sec*

(c) 24 *rad/sec* (d) 36 *rad/sec*

19. A body of mass m hangs at one end of a string of length *l*, the other end of which is fixed. It is given a horizontal velocity so that the string would just reach where it makes an angle of 60^o with the vertical. The tension in the string at mean position is

(a) $2mg$ (b) mg

(c) $3mg$ (d) $\sqrt{3}mg$

20. In a vertical circle of radius *r*, at what point in its path a particle has tension equal to zero if it is just able to complete the vertical circle

(a) Highest point

(b) Lowest point

(c) Any point

(d) At a point horizontally from the centre of circle of radius r

21. The tension in the string revolving in a vertical circle with a mass m at the end which is at the lowest position

[EAMCET (Engg.) 1995; AIIMS 2001]

(a) $\frac{mv^2}{r}$ (b) $\frac{mv^2}{r} - mg$

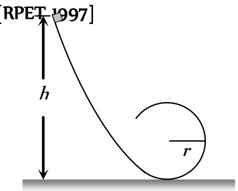
(c) $\frac{mv^2}{r} + mg$ (d) mg

22. A hollow sphere has radius 6.4 *m*. Minimum velocity required by a motor cyclist at bottom to complete the circle will be

(a) 17.7 *m/s* (b) 10.2 *m/s*

(c) 12.4 *m/s* (d) 16.0 *m/s*

23. A block follows the path as shown in the figure from height h. If radius of circular path is r, then relation that holds good to complete full circle is [RPET 1997]



(a) $h < 5r/2$

(b) $h > 5r/2$

(c) $h = 5r/2$

(d) $h \geq 5r/2$

24. A pendulum bob on a 2 *m* string is displaced 60° from the vertical and then released. What is the speed of the bob as it passes through the lowest point in its path [JIPMER 1999]

(a) $\sqrt{2}\ m/s$ (b) $\sqrt{9.8}\ m/s$

(c) 4.43 *m/s* (d) $1/\sqrt{2}\ m/s$

25. A fan is making 600 revolutions per *minute*. If after some time it makes 1200 revolutions per *minute*, then increase in its angular velocity is [BHU 1999]

(a) $10\pi\, rad/\sec$ (b) $20\pi\, rad/\sec$

(c) $40\pi\, rad/\sec$ (d) $60\pi\, rad/\sec$

26. A particle is tied to 20*cm* long string. It performs circular motion in vertical plane. What is the angular velocity of string when the tension in the string at the top is zero

[RPMT 1999]

(a) $5\, rad/sec$ (b) $2\, rad/sec$

(c) $7.5\, rad/sec$ (d) $7\, rad/sec$

27. A stone tied with a string, is rotated in a vertical circle. The minimum speed with which the string has to be rotated

[CBSE PMT 1999]

(a) Is independent of the mass of the stone

(b) Is independent of the length of the string

(c) Decreases with increasing mass of the stone

(d) Decreases with increasing in length of the string

28. For a particle in a non-uniform accelerated circular motion

[ISM Dhanbad 1994] [AMU (Med.) 2000]

(a) Velocity is radial and acceleration is transverse only

(b) Velocity is transverse and acceleration is radial only

(c) Velocity is radial and acceleration has both radial and transverse components

(d) Velocity is transverse and acceleration has both radial and transverse components [EAMCET 1994]

29. A fighter plane is moving in a vertical circle of radius '*r*'. Its minimum velocity at the highest point of the circle will be

[MP PET 2000]

(a) $\sqrt{3gr}$ (b) $\sqrt{2gr}$

(c) $\sqrt{gr}$ (d) $\sqrt{gr/2}$

30. A ball is moving to and fro about the lowest point *A* of a smooth hemispherical bowl. If it is able to rise up to a height of 20 *cm* on either side of *A*, its speed at *A* must be (Take = 10 *m/s*, mass of the body 5 *g*) [JIPMER 2000]

(a) 0.2 *m/s* (b) 2 *m/s*

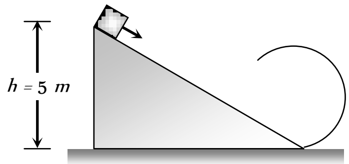
(c) 4 *m/s* (d) 4.5 *ms*

31. A stone of mass *m* is tied to a string and is moved in a vertical circle of radius *r* making *n* revolutions per *minute*. The total tension in the string when the stone is at its lowest point is [RPET 1997]

(a) *mg* (b) $m(g + \pi n r^2)$

(c) $m(g+\pi nr)$ (d) $m\{g+(\pi^2 n^2 r)/900\}$

32. As per given figure to complete the circular loop what should be the radius if initial height is 5 *m* [RPET 2001]



(a) 4 *m*
(b) 3 *m*
(c) 2.5 *m*
(d) 2 *m*

33. A coin, placed on a rotating turn-table slips, when it is placed at a distance of 9 *cm* from the centre. If the angular velocity of the turn-table is trippled, it will just slip, if its distance from the centre is [CPMT 2001]

(a) 27 *cm* (b) 9 *cm*
(c) 3 *cm* (d) 1 *cm*

34. When a ceiling fan is switched off its angular velocity reduces to 50% while it makes 36 rotations. How many more rotation will it make before coming to rest (Assume uniform angular retardation)

(a) 18 (b) 12
(c) 36 (d) 48

35. A body crosses the topmost point of a vertical circle with critical speed. Its centripetal acceleration, when the string is horizontal will be [MH CET 2002]

(a) 6 *g* (b) 3 *g*
(c) 2 *g* (d) *g*

36. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical

(a) 30° (b) 45°
(c) 60° (d) 90°

37. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from vertical (lowest position) are T_1 and T_2 respectively. then

(a) $T_1 = T_2$
(b) $T_2 > T_1$
(c) $T_1 > T_2$
(d) Tension in the string always remains the same

38. A particle is kept at rest at the top of a sphere of diameter 42 *m*. When disturbed slightly, it slides down. At what height '*h*' from the bottom, the particle will leave the sphere [BHU 2003]

(a) 14 *m* (b) 28 *m*
(c) 35 *m* (d) 7 *m*

39. The coordinates of a moving particle at any time '*t*' are given by $x = \alpha t$ and $y = \beta t$. The speed of the particle at time '*t*' is given by

(a) $\sqrt{\alpha^2+\beta^2}$ (b) $3t\sqrt{\alpha^2+\beta^2}$
(c) $3t^2\sqrt{\alpha^2+\beta^2}$ (d) $t^2\sqrt{\alpha^2+\beta^2}$

40. A small disc is on the top of a hemisphere of radius R. What is the smallest horizontal velocity v that should be given to the disc for it to leave the hemisphere and not slide down it ? [There is no friction] [CPMT 1991]

(a) $v=\sqrt{2gR}$ (b) $v=\sqrt{gR}$
(c) $v=\dfrac{g}{R}$ (d) $v=\sqrt{g^2R}$

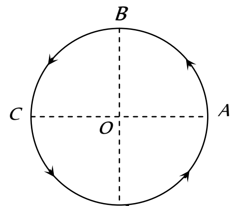
41. A body of mass 0.4 *kg* is whirled in a vertical circle making 2 *rev/sec*. If the radius of the circle is 2 *m*, then tension in the string when the body is at the top of the circle, is [CBSE PMT 1999]

(a) 41.56 *N* (b) 89.86 *N*
(c) 109.86 *N* (d) 115.86 *N*

42. A bucket full of water is revolved in vertical circle of radius 2*m*. What should be the maximum time-period of revolution so that the water doesn't fall off the bucket [AFMC 2004]

(a) 1 *sec* (b) 2 *sec*
(c) 3 *sec* (d) 4 *sec*

43. Figure shows a body of mass *m* moving with a uniform speed *v* along a circle of radius *r*. The change in velocity in going from *A* to *B* is [DPMT 2004]



(a) $v\sqrt{2}$
(b) $v/\sqrt{2}$ [KCET 2001]
(c) v
(d) zero

44. The maximum and minimum tension in the string whirling in a circle of radius 2.5 *m* with constant velocity are in the ratio 5 : 3 then its velocity is [Pb. PET 2003]

(a) $\sqrt{98}\ m/s$ (b) $7\ m/s$
(c) $\sqrt{490}\ m/s$ (d) $\sqrt{4.9}$

45. For a particle in circular motion the centripetal acceleration is [Orissa JEE 2002]

(a) Less than its tangential acceleration
(b) Equal to its tangential acceleration
(c) More than its tangential acceleration
(d) May be more or less than its tangential acceleration

46. A particle moves in a circular path with decreasing speed. Choose the correct statement. [Orissa JEE 2002] [IIT JEE 2005]

(a) Angular momentum remains constant
(b) Acceleration ($\vec{a}$) is towards the center
(c) Particle moves in a spiral path with decreasing radius
(d) The direction of angular momentum remains constant

47. A body of mass 1 *kg* is moving in a vertical circular path of radius 1*m*. The difference between the kinetic energies at its highest and lowest position is

(a) 20*J* (b) 10*J*
(c) $4\sqrt{5}J$ [AIEEE 2003] (d) $10(\sqrt{5}-1)J$

48. The angle turned by a body undergoing circular motion depends on time as $\theta=\theta_0+\theta_1 t+\theta_2 t^2$. Then the angular acceleration of the body is [Orissa JEE 2005]

(a) θ_1 (b) θ_2
(c) $2\theta_1$ (d) $2\theta_2$

Horizontal Projectile Motion

1. The maximum range of a gun on horizontal terrain is 16 *km*. If $g = 10 m/s^2$. What must be the muzzle velocity of the shell

(a) 200 *m/s* (b) 400 *m/s*

(c) 100 *m/s* (d) 50 *m/s*

2. A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following [NCERT 1972; AFMC 1996; BHU 2000]

(a) Straight path (b) Circular path

(c) Parabolic path (d) Hyperbolic path

3. A bullet is dropped from the same height when another bullet is fired horizontally. They will hit the ground

(a) One after the other (b) Simultaneously

(c) Depends on the observer (d) None of the above

4. An aeroplane is flying at a constant horizontal velocity of 600 *km/hr* at an elevation of 6 *km* towards a point directly above the target on the earth's surface. At an appropriate time, the pilot releases a ball so that it strikes the target at the earth. The ball will appear to be falling [MP PET 1993]

(a) On a parabolic path as seen by pilot in the plane

(b) Vertically along a straight path as seen by an observer on the ground near the target

(c) On a parabolic path as seen by an observer on the ground near the target

(d) On a zig-zag path as seen by pilot in the plane

5. A bomb is dropped from an aeroplane moving horizontally at constant speed. When air resistance is taken into consideration, the bomb

[EAMCET (Med.) 1995; AFMC 1999]

(a) Falls to earth exactly below the aeroplane

(b) Fall to earth behind the aeroplane

(c) Falls to earth ahead of the aeroplane

(d) Flies with the aeroplane

6. A man projects a coin upwards from the gate of a uniformly moving train. The path of coin for the man will be

[RPET 1997]

(a) Parabolic (b) Inclined straight line

(c) Vertical straight line (d) Horizontal straight line

7. An aeroplane is flying horizontally with a velocity of 600 *km/h* at a height of 1960 *m*. When it is vertically at a point A on the ground, a bomb is released from it. The bomb strikes the ground at point *B*. The distance *AB* is

[CPMT 1996; JIPMER 2001, 02]

(a) 1200 *m* (b) 0.33 *km*

(c) 3.33 *km* (d) 33 *km*

8. A ball is rolled off the edge of a horizontal table at a speed of 4 *m/second*. It hits the ground after 0.4 *second*. Which statement given below is true [AMU (Med.) 1999]

(a) It hits the ground at a horizontal distance 1.6 *m* from the edge of the table

(b) The speed with which it hits the ground is 4.0 *m/second*

(c) Height of the table is 0.8 *m*

(d) It hits the ground at an angle of 60° to the horizontal

9. An aeroplane flying 490 *m* above ground level at 100 *m/s*, releases a block. How far on ground will it strike

[RPMT 2000]

(a) 0.1 *km* (b) 1 *km*

(c) 2 *km* (d) None

10. A body is thrown horizontally from the top of a tower of height 5 *m*. It touches the ground at a distance of 10 *m* from the foot of the tower. The initial velocity of the body is (g = 10 ms^{-2})

(a) 2.5 ms^{-1} (b) 5 ms^{-1}

(c) 10 ms^{-1} (d) 20 ms^{-1}

11. An aeroplane moving horizontally with a speed of 720 *km/h* drops a food pocket, while flying at a height of 396.9 *m*. the time taken by a food pocket to reach the ground and its horizontal range is (Take g = 9.8 *m/sec*) [AFMC 2001]

(a) 3 *sec* and 2000 *m* (b) 5 *sec* and 500 *m*

(c) 8 *sec* and 1500 *m* (d) 9 *sec* and 1800 *m*

12. A particle (*A*) is dropped from a height and another particle (*B*) is thrown in horizontal direction with speed of 5 *m/sec* from the same height. The correct statement is

[CBSE PMT 2002; Orissa JEE 2003]

(a) Both particles will reach at ground simultaneously

(b) Both particles will reach at ground with same speed

(c) Particle (*A*) will reach at ground first with respect to particle (*B*)

(d) Particle (*B*) will reach at ground first with respect to particle (*A*)

13. A particle moves in a plane with constant acceleration in a direction different from the initial velocity. The path of the particle will be [MP PMT 2004

(a) A straight line (b) An arc of a circle

(c) A parabola (d) An ellipse

14. At the height 80 *m*, an aeroplane is moving with 150 *m/s*. A bomb is dropped from it so as to hit a target. At what distance from the target should the bomb be dropped (given g = 10 *m/s*)

(a) 605.3 *m* (b) 600 *m*

(c) 80 *m* (d) 230 *m*

15. A bomber plane moves horizontally with a speed of 500 *m/s* and a bomb released from it, strikes the ground in 10 sec. Angle at which it strikes the ground will be $(g = 10\ m/s^2)$

[MH CET 2003]

(a) $\tan^{-1}\left(\frac{1}{5}\right)$ (b) $\tan\left(\frac{1}{5}\right)$

(c) $\tan^{-1}(1)$ (d) $\tan^{-1}(5)$

16. A large number of bullets are fired in all directions with same speed v. What is the maximum area on the ground on which these bullets will spread

(a) $\pi\frac{v^2}{g}$ (b) $\pi\frac{v^4}{g^2}$

(c) $\pi^2\frac{v^4}{g^2}$ (d) $\pi^2\frac{v^2}{g^2}$

Oblique Projectile Motion

1. A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be

(a) $2R$ (b) $R/2$

(c) R (d) $4R$

2. If the initial velocity of a projectile be doubled, keeping the angle of projection same, the maximum height reached by it will

(a) Remain the same (b) Be doubled

(c) Be quadrupled (d) Be halved

3. In the motion of a projectile freely under gravity, its

(a) Total energy is conserved

(b) Momentum is conserved

(c) Energy and momentum both are conserved

(d) None is conserved

4. The range of a projectile for a given initial velocity is maximum when the angle of projection is 45^o. The range will be minimum, if the angle of projection is

(a) 90^o (b) 180^o

(c) 60^o (d) 75^o

5. The angle of projection at which the horizontal range and maximum height of projectile are equal is

[Kurukshetra CEE 1996; BCECE 2003; Pb. PET 2001]

(a) 45^o (b) $\theta = \tan^{-1}(0.25)$

(c) $\theta = \tan^{-1} 4$ or $(\theta = 76^o)$ (d) 60^o

6. A ball is thrown upwards and it returns to ground describing a parabolic path. Which of the following remains constant

[BHU 1999; DPMT 2001; AMU (Engg.) 2000]

(a) Kinetic energy of the ball

(b) Speed of the ball

(c) Horizontal component of velocity

(d) Vertical component of velocity

7. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are

(a) Perpendicular to each other

(b) Parallel to each other

(c) Inclined to each other at an angle of 45^o

(d) Antiparallel to each other

8. An object is thrown along a direction inclined at an angle of 45^o with the horizontal direction. The horizontal range of the particle is equal to [MP PMT 1985]

(a) Vertical height

(b) Twice the vertical height

(c) Thrice the vertical height

(d) Four times the vertical height

9. The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = (8t - 5t^2)$ *meter* and $x = 6t$ *meter*, where t is in second. The velocity with which the projectile is projected is[CPMT 1981; MP PET

(a) 8 *m/sec*

(b) 6 *m/sec*

(c) 10 *m/sec*

(d) Not obtainable from the data

10. Referring to above question, the angle with the horizontal at which the projectile was projected is [CPMT 1981]

(a) $\tan^{-1}(3/4)$

(b) $\tan^{-1}(4/3)$

(c) $\sin^{-1}(3/4)$

(d) Not obtainable from the given data

11. Referring to the above two questions, the acceleration due to gravity is given by [CPMT 1981]

(a) $10\, m/\sec^2$ (b) $5m/\sec^2$

(c) $20\ m/\sec^2$ (d) $2.5\ m/\sec^2$

12. The range of a particle when launched at an angle of 15^o with the horizontal is 1.5 *km*. What is the range of the projectile when launched at an angle of 45^o to the horizontal

(a) 1.5 *km* (b) 3.0 *km*

(c) 6.0 *km* (d) 0.75 *km*

13. A cricketer hits a ball with a velocity $25\ m/s$ at 60^o above the horizontal. How far above the ground it passes over a fielder 50 *m* from the bat (assume the ball is struck very close to the ground)

(a) 8.2 *m* (b) 9.0 *m*

(c) 11.6 *m* (d) 12.7 *m*

14. A stone is projected from the ground with velocity $25\, m/s$. Two seconds later, it just clears a wall 5 *m* high. The angle of projection of the stone is $(g = 10 m/\sec^2)$

(a) 30^o (b) 45^o

(c) 50.2^o (d) 60^o

15. Galileo writes that for angles of projection of a projectile at angles $(45 + \theta)$ and $(45 - \theta)$, the horizontal ranges described by the projectile are in the ratio of (if $\theta \le 45$)

[MP PET 1993]

(a) 2 : 1 (b) 1 : 2

(c) 1 : 1 (d) 2 : 3

16. A projectile thrown with a speed v at an angle θ has a range R on the surface of earth. For same v and θ, its range on the surface of moon will be

(a) $R/6$ (b) $6R$

(c) $R/36$ (d) $36R$

17. The greatest height to which a man can throw a stone is h. The greatest distance to which he can throw it, will be

(a) $\frac{h}{2}$ (b) h

(c) $2h$ (d) $3h$

18. The horizontal range is four times the maximum height attained by a projectile. The angle of projection is

[MP PET 1994; CBSE PMT 2000; RPET 2001]

(a) 90^o (b) 60^o

(c) 45^o (d) 30^o

19. A ball is projected with kinetic energy E at an angle of 45^o to the horizontal. At the highest point during its flight, its kinetic energy will be

[MP PMT 1994; CBSE PMT 1997, 2001; AIEEE 2002; Pb. PMT 2004; Orissa PMT 2004]

(a) Zero (b) $\frac{E}{2}$

(c) $\frac{E}{\sqrt{2}}$ (d) E

20. A particle of mass m is projected with velocity v making an angle of 45^o with the horizontal. The magnitude of the angular momentum of the particle about the point of projection when the particle is at its maximum height is (where g = acceleration due to gravity)

[MP PMT 1994; MP PET 2001; Pb. PET 2004]

(a) Zero (b) $mv^3/(4\sqrt{2}g)$

(c) $mv^3/(\sqrt{2}g)$ (d) $mv^2/2g$

21. A particle reaches its highest point when it has covered exactly one half of its horizontal range. The corresponding point on the displacement time graph is characterised by

[AIIMS 1995]

(a) Negative slope and zero curvature

(b) Zero slope and negative curvature

(c) Zero slope and positive curvature

(d) Positive slope and zero curvature

22. At the top of the trajectory of a projectile, the acceleration is

(a) Maximum (b) Minimum

(c) Zero (d) g

23. When a body is thrown with a velocity u making an angle θ with the horizontal plane, the maximum distance covered by it in horizontal direction is [MP PMT 1996; RPET 2001]

(a) $\frac{u^2 \sin\theta}{g}$ (b) $\frac{u^2 \sin 2\theta}{2g}$

(c) $\frac{u^2 \sin 2\theta}{g}$ (d) $\frac{u^2 \cos 2\theta}{g}$

24. A football player throws a ball with a velocity of 50 *metre/sec* at an angle 30 degrees from the horizontal. The ball remains in the air for $(g = 10\, m/s^2)$

(a) 2.5 *sec* (b) 1.25 *sec*

(c) 5 *sec* (d) 0.625 *sec*

25. A body of mass 0.5 *kg* is projected under gravity with a speed of 98 *m/s* at an angle of 30^o with the horizontal. The change in momentum (in magnitude) of the body is

[MP PET 1997]

(a) $24.5\, N-s$ (b) $49.0\, N-s$

(c) $98.0\, N-s$ (d) $50.0\, N-s$

26. A body is projected at such an angle that the horizontal range is three times the greatest height. The angle of projection is [AIIM

(a) $25^o 8'$ (b) $33^o 7'$

(c) $42^o 8'$ (d) $53^o 8'$

27. A gun is aimed at a target in a line of its barrel. The target is released and allowed to fall under gravity at the same instant the gun is fired. The bullet will [EAMCET 1994]

(a) Pass above the target (b) Pass below the target

(c) Hit the target (d) Certainly miss the target

28. Two bodies are projected with the same velocity. If one is projected at an angle of 30^o and the other at an angle of 60^o to the horizontal, the ratio of the maximum heights reached is

[AIIMS 2001; EAMCET (Med.) 1995; Pb. PMT 2000]

(a) 3 : 1 (b) 1 : 3

(c) 1 : 2 (d) 2 : 1

29. If the range of a gun which fires a shell with muzzle speed V is R, then the angle of elevation of the gun is [AMU 1995]

(a) $\cos^{-1}\left(\frac{V^2}{Rg}\right)$ (b) $\cos^{-1}\left(\frac{gR}{V^2}\right)$

(c) $\frac{1}{2}\left(\frac{V^2}{Rg}\right)$ (d) $\frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$

30. If time of flight of a projectile is 10 seconds. Range is 500 *meters*. The maximum height attained by it will be

[RPMT 1997]

(a) 125 *m* (b) 50 *m*

(c) 100 *m* (d) 150 *m*

31. If a body A of mass M is thrown with velocity V at an angle of 30^o to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60^o to the horizontal. The ratio of horizontal range of A to B will be

[CBSE PMT 1992]

(a) 1 : 3 [Manipal MEE 1995] (b) 1 : 1

(c) $1:\sqrt{3}$ (d) $\sqrt{3}:1$

32. A bullet is fired from a cannon with velocity 500 *m/s*. If the angle of projection is 15^o and $g = 10 m/s^2$. Then the range is

(a) $25 \times 10^3 m$ (b) $12.5 \times 10^3 m$

(c) $50 \times 10^2 m$ (d) $25 \times 10^2 m$

33. A ball thrown by a boy is caught by another after 2 *sec*. some distance away in the same level. If the angle of projection is 30°, the velocity of projection is [JIPMER 1999]

(a) 19.6 *m/s* (b) 9.8 *m/s*

(c) 14.7 *m/s* (d) None of these

34. A particle covers 50 *m* distance when projected with an initial speed. On the same surface it will cover a distance, when projected with double the initial speed [RPMT 2000]

(a) 100 *m* (b) 150 *m*

(c) 200 *m* (d) 250 *m*

35. A ball is thrown upwards at an angle of 60° to the horizontal. It falls on the ground at a distance of 90 *m*. If the ball is thrown with the same initial velocity at an angle 30°, it will fall on the ground at a distance of [BHU 2000]

(a) 30 *m* (b) 60 *m*

(c) 90 *m* (d) 120 *m*

36. Four bodies *P, Q, R* and *S* are projected with equal velocities having angles of projection 15°, 30°, 45° and 60° with the horizontal respectively. The body having shortest range is

(a) *P* (b) *Q*

(c) *R* (d) *S*

37. For a projectile, the ratio of maximum height reached to the square of flight time is (g = 10 ms^{-2})

[EAMCET (Med.) 2000]

(a) 5 : 4 (b) 5 : 2

(c) 5 : 1 (d) 10 : 1

38. A stone projected with a velocity u at an angle θ with the horizontal reaches maximum height H_1. When it is projected with velocity u at an angle $\left(\frac{\pi}{2}-\theta\right)$ with the horizontal, it reaches maximum height H_2. The relation between the horizontal range R of the projectile, H_1 and H_2 is

(a) $R=4\sqrt{H_1H_2}$ (b) $R=4(H_1-H_2)$

(c) $R=4(H_1+H_2)$ (d) $R=\frac{H_1^2}{H_2^2}$

39. An object is projected with a velocity of 20 *m/s* making an angle of 45° with horizontal. The equation for the trajectory is $h = Ax - Bx^2$ where h is height, x is horizontal distance, A and B are constants. The ratio $A : B$ is (g = 10 ms^{-2})

[EAMCET 2001]

(a) 1 : 5 (b) 5 : 1

(c) 1 : 40 (d) 40 : 1

40. Which of the following sets of factors will affect the horizontal distance covered by an athlete in a long–jump event

(a) Speed before he jumps and his weight

(b) The direction in which he leaps and the initial speed

(c) The force with which he pushes the ground and his speed

(d) None of these

41. A ball thrown by one player reaches the other in 2 *sec.* the maximum height attained by the ball above the point of projection will be about [Pb. PMT 2002]

(a) 10 *m* (b) 7.5 *m*

(c) 5 *m* (d) 2.5 *m*

42. In a projectile motion, velocity at maximum height is

[AIEEE 2002]

(a) $\frac{u\cos\theta}{2}$ (b) $u\cos\theta$

(c) $\frac{u\sin\theta}{2}$ (d) None of these

43. If two bodies are projected at 30° and 60° respectively, with the same velocity, then [JIPMER 2002; CBSE PMT 2000]

(a) Their ranges are same

(b) Their heights are same

(c) Their times of flight are same

(d) All of these

44. A body is thrown with a velocity of 9.8 *m/s* making an angle of 30° with the horizontal. It will hit the ground after a time

[JIPMER 2001, 2002; KCET 2001]

(a) 1.5 *s* (b) 1 *s*

(c) 3 *s* (d) 2 *s*

45. The equation of motion of a projectile are given by x = 36 t *metre* and $2y$ = 96 t – 9.8 t^2 *metre*. The angle of projection is

(a) $\sin^{-1}\left(\frac{4}{5}\right)$ (b) $\sin^{-1}\left(\frac{3}{5}\right)$

[EAMCET (Engg.) 2000]

(c) $\sin^{-1}\left(\frac{4}{3}\right)$ (d) $\sin^{-1}\left(\frac{3}{4}\right)$

46. For a given velocity, a projectile has the same range R for two angles of projection if t_1 and t_2 are the times of flight in the two cases then[KCET 2003;

(a) $t_1t_2\propto R^2$ (b) $t_1t_2\propto R$

(c) $t_1t_2\propto\frac{1}{R}$ (d) $t_1t_2\propto\frac{1}{R^2}$

47. A body of mass m is thrown upwards at an angle θ with the horizontal with velocity v. While rising up the velocity of the mass after t *seconds* will be [AMU (Engg.) 1999]

(a) $\sqrt{(v\cos\theta)^2+(v\sin\theta)^2}$

[EAMCET 2000]

(b) $\sqrt{(v\cos\theta-v\sin\theta)^2-gt}$

(c) $\sqrt{v^2+g^2t^2-(2v\sin\theta)gt}$

(d) $\sqrt{v^2+g^2t^2-(2v\cos\theta)gt}$

48. A cricketer can throw a ball to a maximum horizontal distance of 100 *m*. With the same effort, he throws the ball vertically upwards. The maximum height attained by the ball is

(a) 100 *m* (b) 80 *m*

(c) 60 *m* (d) 50 *m*

49. A cricketer can throw a ball to a maximum horizontal distance of 100 *m*. The speed with which he throws the ball is (to the nearest integer) [Kerala (Med.) 2002]

[AMU (Engg.) 2001]

(a) 30 ms^{-1} (b) 42 ms^{-1}

(c) 32 ms^{-1} (d) 35 ms^{-1}

50. A ball is projected with velocity V_o at an angle of elevation 30°. Mark the correct statement [MP PMT 2004]

(a) Kinetic energy will be zero at the highest point of the trajectory

(b) Vertical component of momentum will be conserved

(c) Horizontal component of momentum will be conserved

(d) Gravitational potential energy will be minimum at the highest point of the trajectory

51. Neglecting the air resistance, the time of flight of a projectile is determined by [J & K CET 2004]

(a) $U_{vertical}$

(b) $U_{horizontal}$

(c) $U=U^2{}_{vertical}+U^2{}_{horizontal}$

(d) $U=U(U^2{}_{vertical}+U^2{}_{horizontal})^{1/2}$

52. A ball is thrown from a point with a speed v_o at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_o/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection [AIEEE 2004]

(a) Yes, 60^o (b) Yes, 30^o

(c) No (d) Yes, 45^o

53. A stone is thrown at an angle θ to the horizontal reaches a maximum height *H*. Then the time of flight of stone will be [BCECE 2004]

(a) $\sqrt{\dfrac{2H}{g}}$ (b) $2\sqrt{\dfrac{2H}{g}}$

(c) $\dfrac{2\sqrt{2H\sin\theta}}{g}$ (d) $\dfrac{\sqrt{2H\sin\theta}}{g}$

54. The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Its angle of projection will be [J & K CET 2004; DPMT 2003]

(a) 45^o (b) 60^o

(c) 90^o (d) 30^o

55. A ball is projected upwards from the top of tower with a velocity $50\ ms^{-1}$ making an angle 30^o with the horizontal. The height of tower is 70 *m*. After how many seconds from the instant of throwing will the ball reach the ground [DPMT 2004]

(a) 2 *s* (b) 5 *s*

(c) 7 *s* (d) 9 *s*

56. Two bodies are thrown up at angles of 45° and 60°, respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is

(a) $\sqrt{\dfrac{2}{3}}$ (b) $\dfrac{2}{\sqrt{3}}$

(c) $\sqrt{\dfrac{3}{2}}$ (d) $\dfrac{\sqrt{3}}{2}$

57. At what point of a projectile motion acceleration and velocity are perpendicular to each other [Orissa JEE 2005]

(a) At the point of projection

(b) At the point of drop

(c) At the topmost point

(d) Any where in between the point of projection and topmost point

58. An object is projected at an angle of 45° with the horizontal. The horizontal range and the maximum height reached will be in the ratio. [Kerala PET 2005]

(a) 1 : 2 (b) 2 : 1

(c) 1 : 4 (d) 4 : 1

59. The maximum horizontal range of a projectile is 400 *m*. The maximum value of height attained by it will be [AFMC 2005]

(a) 100 *m* (b) 200 *m*

(c) 400 *m* (d) 800 *m*

Critical Thinking

Objective Questions

1. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

(a) Velocity is constant

(b) Acceleration is constant

(c) Kinetic energy is constant

(d) It moves in a circular path

2. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is [IIT 1992]

(a) $\dfrac{ML\omega^2}{2}$ (b) $ML\omega^2$

(c) $\dfrac{ML\omega^2}{4}$ (d) $\dfrac{ML^2\omega^2}{2}$

3. The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered s as $k = as^2$ where a is a constant. The force acting on the particle is [MNR 1992; JIPMER 2001, 02; AMU (Engg.) 1999]

(a) $2a\dfrac{s^2}{R}$ (b) $2as\left(1+\dfrac{s^2}{R^2}\right)^{1/2}$

(c) $2as$ [DPMT 2005] (d) $2a\dfrac{R^2}{s}$

4. A car is moving in a circular horizontal track of radius 10 *m* with a constant speed of 10 *m/sec*. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1.00 *m*. The angle made by the rod with track is [IIT 1992]

(a) Zero (b) 30^o

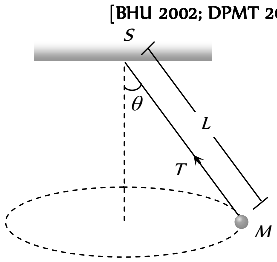
(c) 45^o (d) 60^o

5. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as, $a_c = k^2rt^2$, The power delivered to the particle by the forces acting on it is [IIT 1994]

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t

(c) $\dfrac{mk^4r^2t^5}{3}$ (d) Zero

6. A string of length *L* is fixed at one end and carries a mass *M* at the other end. The string makes $2/\pi$ revolutions per *second* around the vertical axis through the fixed end as shown in the figure, then tension in the string is [BHU 2002; DPMT 2004]



(a) *ML*

(b) 2 *ML*

(c) 4 *ML*

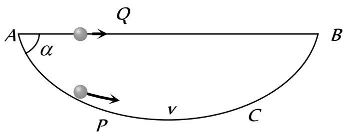
(d) 16 *ML*

7. A stone of mass 1 *kg* tied to a light inextensible string of length $L = \dfrac{10}{3}\,m$ is whirling in a circular path of radius L in a vertical plane. [IIT 1987] The ratio of the maximum tension in the string to the minimum tension in the string is 4 and if g is taken to be $10 m/\sec^2$, the speed of the stone at the highest point of the circle is [CBSE PMT 1990]

(a) $20 m/\sec$ (b) $10\sqrt{3} m/\sec$

(c) $5\sqrt{2}\ m/\sec$ (d) $10 m/\sec$

8. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at $t = 0$ along the horizontal string AB (see figure) with the speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective time taken by P and Q to reach the point B. Then



(a) $t_P < t_Q$

(b) $t_P = t_Q$

(c) $t_P > t_Q$

(d) All of these

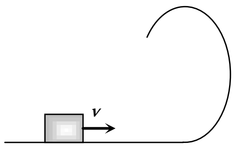
9. A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

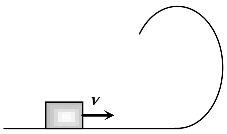
[IIT-JEE Screening 2000]

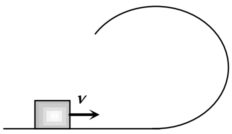
(a) $\sqrt{\dfrac{\mu}{\alpha}}$ (b) $\dfrac{\mu}{\sqrt{\alpha}}$

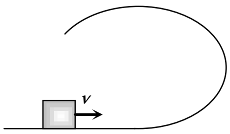
(c) $\dfrac{1}{\sqrt{\mu\alpha}}$ (d) Infinitesimal

10. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in

(a) 

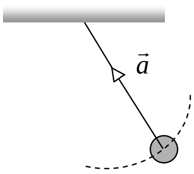
(b) 

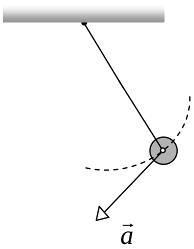
(c) 

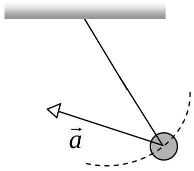
(d) 

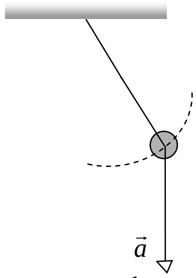
11. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in

[IIT-JEE Screening 2002]

(a) 

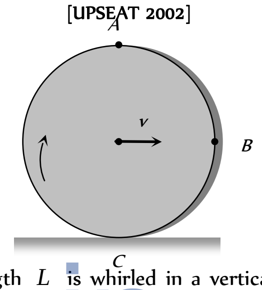
(b) 

(c) 

(d) 

[IIT 1993]

12. A solid disc rolls clockwise without slipping over a horizontal path with a constant speed υ. Then the magnitude of the velocities of points A, B and C (see figure) with respect to a standing observer are respectively [UPSEAT 2002]



(a) υ, υ and υ

(b) $2\upsilon, \sqrt{2}\upsilon$ and zero

(c) $2\upsilon, 2\upsilon$ and zero

(d) $2\upsilon, \sqrt{2}\upsilon$ and $\sqrt{2}\upsilon$

13. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is

[IIT 1998; CBSE PMT 2004]

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gl}$ (d) $\sqrt{2(u^2 - gL)}$

14. The driver of a car travelling at velocity v suddenly see a broad wall in front of him at a distance d. He should

[IIT 1977]

[IIT-JEE Screening 2001]

(a) Brake sharply (b) Turn sharply

(c) (a) and (b) both (d) None of the above

15. Four persons K, L, M and N are initially at the corners of a square of side of length d. If every person starts moving, such that K is always headed towards L, L towards M, M is headed directly towards N and N towards K, then the four persons will meet after [IIT 1984]

(a) $\dfrac{d}{v}$ sec (b) $\dfrac{\sqrt{2}d}{v}$ sec

(c) $\dfrac{d}{\sqrt{2}v}$ sec (d) $\dfrac{d}{2v}$ sec

16. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$ where $a, b (< a)$ and p are positive constants of appropriate dimensions. Then

[IIT-JEE 1999]

(a) The path of the particle is an ellipse

(b) The velocity and acceleration of the particle are normal to each other at $t = \pi/(2p)$

(c) The acceleration of the particle is always directed towards a focus

(d) The distance travelled by the particle in time interval $t = 0$ to $t = \pi/(2p)$ is a

17. A particle is moving eastwards with velocity of 5 m/s. In 10 sec the velocity changes to 5 m/s northwards. The average acceleration in this time is

[IIT 1982; AFMC 1999; Pb PET 2000; JIPMER 2001, 02]

(a) Zero

(b) $\frac{1}{\sqrt{2}}\ m/s^2$ toward north-west

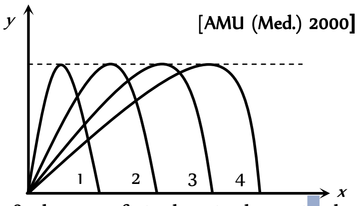
(c) $\frac{1}{\sqrt{2}}\ m/s^2$ toward north-east

(d) $\frac{1}{2}\ m/s^2$ toward north-west

GQ Graphical Questions

1. Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component, highest first

[AMU (Med.) 2000]



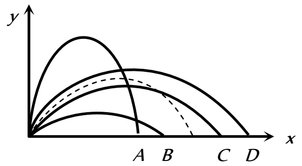
(a) 1, 2, 3, 4

(b) 2, 3, 4, 1

(c) 3, 4, 1, 2

(d) 4, 3, 2, 1

2. The path of a projectile in the absence of air drag is shown in the figure by dotted line. If the air resistance is not ignored then which one of the path shown in the figure is appropriate for the projectile



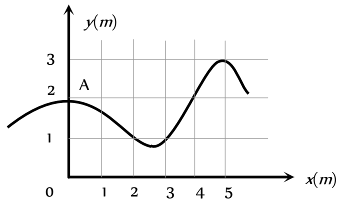
(a) B

(b) A

(c) D

(d) C

3. The trajectory of a particle moving in vast maidan is as shown in the figure. The coordinates of a position A are (0,2). The coordinates of another point at which the instantaneous velocity is same as the average velocity between the points are



(a) (1, 4)

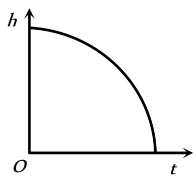
(b) (5, 3)

(c) (3, 4)

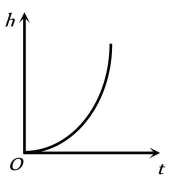
(d) (4, 1)

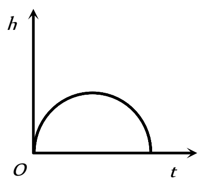
4. Which of the following is the graph between the height (h) of a projectile and time (t), when it is projected from the ground

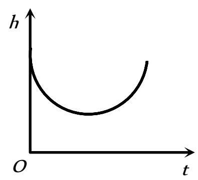
(a)



(b)





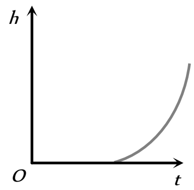


(c)

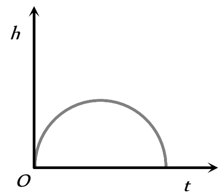
(d)

5. Which of the following is the altitude-time graph for a projectile thrown horizontally from the top of the tower

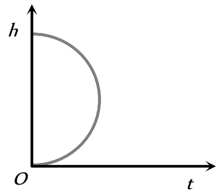
(a)



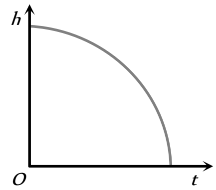
(b)



(c)



(d)



AR Assertion & Reason

For AIIMS Aspirants

Read the assertion and reason carefully to mark the correct option out of the options given below:

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

(c) If assertion is true but reason is false.

(d) If the assertion and reason both are false.

(e) If assertion is false but reason is true.

1. Assertion : In projectile motion, the angle between the instantaneous velocity and acceleration at the highest point is 180°.

Reason : At the highest point, velocity of projectile will be in horizontal direction only.

2. Assertion : Two particles of different mass, projected with same velocity at same angles. The maximum height attained by both the particle will be same.

Reason : The maximum height of projectile is independent of particle mass.

3. Assertion : The maximum horizontal range of projectile is proportional to square of velocity.

Reason : The maximum horizontal range of projectile is equal to maximum height attained by projectile.

4. Assertion : Horizontal range is same for angle of projection θ and $(90 - \theta)$.

Reason : Horizontal range is independent of angle of projection.

5. Assertion : For projection angle $\tan^{-1}(4)$, the horizontal range and the maximum height of a projectile are equal.

	Reason	:	The maximum range of projectile is directly proportional to square of velocity and inversely proportional to acceleration due to gravity.
6.	Assertion	:	The trajectory of projectile is quadratic in y and linear in x.
	Reason	:	y component of trajectory is independent of x-component.
7.	Assertion	:	In javelin throw, the athlete throws the projectile at an angle slightly more than 45°.
	Reason	:	The maximum range does not depends upon angle of projection.
8.	Assertion	:	When a body is dropped or thrown horizontally from the same height, it would reach the ground at the same time.
	Reason	:	Horizontal velocity has no effect on the vertical direction.
9.	Assertion	:	When the velocity of projection of a body is made n times, its time of flight becomes n times.
	Reason	:	Range of projectile does not depend on the initial velocity of a body.
10.	Assertion	:	The height attained by a projectile is twenty five percent of range, when projected for maximum range.
	Reason	:	The height is independent of initial velocity of projectile.
11.	Assertion	:	When range of a projectile is maximum, its angle of projection may be 45° or 135°.
	Reason	:	Whether θ is 45° or 135°, value of range remains the same, only the sign changes.
12.	Assertion	:	In order to hit a target, a man should point his rifle in the same direction as target.
	Reason	:	The horizontal range of the bullet is dependent on the angle of projectile with horizontal direction.
13.	Assertion	:	When a particle moves in a circle with a uniform speed, its velocity and acceleration both changes.
	Reason	:	The centripetal acceleration in circular motion is dependent on angular velocity of the body.
14.	Assertion	:	During a turn, the value of centripetal force should be less than the limiting frictional force.
	Reason	:	The centripetal force is provided by the frictional force between the tyres and the road.
15.	Assertion	:	When a vehicle takes a turn on the road, it travels along a nearly circular path.
	Reason	:	In circular motion, velocity of vehicle remains same.
16.	Assertion	:	As the frictional force increases, the safe velocity limit for taking a turn on an unbanked road also increases.
	Reason	:	Banking of roads will increase the value of limiting velocity.
17.	Assertion	:	If the speed of a body is constant, the body cannot have a path other than a circular or straight line path.
	Reason	:	It is not possible for a body to have a constant speed in an accelerated motion.
18.	Assertion	:	In circular motion, work done by centripetal force is zero.
	Reason	:	In circular motion centripetal force is perpendicular to the displacement.
19.	Assertion	:	In circular motion, the centripetal and centrifugal force acting in opposite direction balance each other.
	Reason	:	Centripetal and centrifugal forces don't act at the same time.
20.	Assertion	:	If both the speed of a body and radius of its circular path are doubled, then centripetal force also gets doubled.
	Reason	:	Centripetal force is directly proportional to both speed of a body and radius of circular path.
21.	Assertion	:	When an automobile while going too fast around a curve overturns, its inner wheels leave the ground first.
	Reason	:	For a safe turn the velocity of automobile should be less than the value of safe limit velocity.
22.	Assertion	:	A safe turn by a cyclist should neither be fast nor sharp.
	Reason	:	The bending angle from the vertical would decrease with increase in velocity.
23.	Assertion	:	Improper banking of roads causes wear and tear of tyres.
	Reason	:	The necessary centripetal force is provided by the force of friction between the tyres and the road.
24.	Assertion	:	Cream gets separated out of milk when it is churned, it is due to gravitational force.
	Reason	:	In circular motion gravitational force is equal to centripetal force.
25.	Assertion	:	Two similar trains are moving along the equatorial line with the same speed but in opposite direction. They will exert equal pressure on the rails.
	Reason	:	In uniform circular motion the magnitude of acceleration remains constant but the direction continuously changes.
26.	Assertion	:	A coin is placed on phonogram turn table. The motor is started, coin moves along the moving table.
	Reason	:	The rotating table is providing necessary centripetal force to the coin.

Answers

Uniform Circular Motion

1	c	2	c	3	b	4	b	5	c
6	c	7	c	8	c	9	b	10	b
11	a	12	a	13	c	14	ac	15	a
16	d	17	a	18	a	19	d	20	a
21	b	22	a	23	c	24	d	25	a
26	c	27	b	28	a	29	d	30	c
31	b	32	d	33	d	34	b	35	c
36	d	37	c	38	b	39	a	40	c
41	d	42	b	43	b	44	b	45	d
46	a	47	b	48	a	49	d	50	b

51	a	52	c	53	a	54	b	55	a
56	d	57	a	58	d	59	a	60	b
61	d	62	d	63	a	64	b	65	d
66	d	67	c	68	a	69	c	70	a
71	b	72	d	73	a	74	c	75	b
76	a	77	c	78	a	79	b	80	a
81	c	82	a	83	d	84	c	85	b
86	a	87	c	88	c	89	d	90	b
91	a	92	d	93	a	94	b	95	b
96	c	97	d	98	d	99	a	100	b
101	c	102	c	103	b	104	c	105	d
106	d	107	b	108	a	109	a	110	b
111	b	112	b	113	b	114	a	115	d
116	b	117	d	118	b	119	c	120	d
121	a	122	d						

Non-uniform Circular Motion

1	d	2	d	3	d	4	c	5	a
6	c	7	a	8	d	9	b	10	b
11	b	12	b	13	a	14	b	15	b
16	a	17	c	18	c	19	a	20	a
21	c	22	a	23	d	24	c	25	b
26	d	27	a	28	d	29	c	30	b
31	d	32	d	33	d	34	b	35	b
36	d	37	c	38	c	39	c	40	b
41	d	42	c	43	a	44	a	45	d
46	d	47	a	48	d				

Horizontal Projectile Motion

1	b	2	c	3	b	4	c	5	b
6	c	7	c	8	ac	9	b	10	c
11	d	12	a	13	c	14	a	15	a
16	b								

Oblique Projectile Motion

1	d	2	c	3	a	4	a	5	c
6	c	7	a	8	d	9	c	10	b
11	a	12	b	13	a	14	a	15	c
16	b	17	c	18	c	19	b	20	b
21	b	22	d	23	c	24	c	25	b
26	d	27	c	28	b	29	d	30	a
31	b	32	b	33	a	34	c	35	c
36	a	37	a	38	a	39	d	40	b

41	c	42	b	43	a	44	b	45	a
46	b	47	c	48	d	49	c	50	c
51	a	52	a	53	b	54	d	55	c
56	c	57	c	58	d	59	b		

Critical Thinking Questions

1	cd	2	a	3	b	4	c	5	b
6	d	7	d	8	a	9	a	10	a
11	c	12	b	13	d	14	a	15	a
16	ab	17	b						

Graphical Questions

1	d	2	a	3	b	4	c	5	d

Assertion and Reason

1	e	2	a	3	c	4	c	5	b
6	d	7	d	8	a	9	c	10	c
11	a	12	e	13	b	14	a	15	c
16	b	17	d	18	a	19	d	20	c
21	b	22	c	23	a	24	d	25	e
26	d								

Answers and Solutions

Uniform Circular Motion

1. (c) $v = r\omega \Rightarrow \omega = \frac{v}{r}$ = constant [As v and r are constant]

2. (c) As time periods are equal therefore ratio of angular speeds will be same. $\omega = \frac{2\pi}{T}$

3. (b) $F = \frac{mv^2}{r} \Rightarrow F \propto v^2$. If v becomes double then F (tendency to overturn) will become four times.

4. (b) Work done by centripetal force is always zero.

5. (c) It is always directed in a direction of tangent to circle.

6. (c) Stone flies in the direction of instantaneous velocity due to inertia

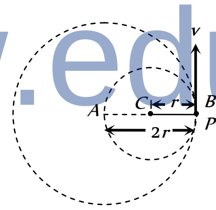
7. (c) Centripetal acceleration $= \frac{v^2}{r}$ = constant. Direction keeps changing.

8. (c) Linear velocity, acceleration and force varies in direction.

9. (b) Angular velocity of particle P about point A,

$\omega_A = \frac{v}{r_{AB}} = \frac{v}{2r}$

Angular velocity of particle P about point C,



$\omega_C = \frac{v}{r_{BC}} = \frac{v}{r}$

Ratio $\frac{\omega_A}{\omega_C} = \frac{v/2r}{v/r} = \frac{1}{2}$.

10. (b)

11. (a) $F = \frac{mv^2}{r}$. If m and v are constants then $F \propto \frac{1}{r}$

$\therefore \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)$

12. (a) In uniform circular motion (constant angular velocity) kinetic energy remains constant but due to change in velocity of particle its momentum varies.

13. (c)

14. (a,c) Centripetal force $= \frac{mv^2}{r}$ and is directed always towards the centre of circle. Sense of rotation does not affect magnitude and direction of this centripetal force.

15. (a) When speed is constant in circular motion, it means work done by centripetal force is zero.

16. (d)

17. (a) This horizontal inward component provides required centripetal force.

18. (a) Thrust at the lowest point of concave bridge

$= mg + \frac{mv^2}{r}$

19. (d)

20. (a) Because the reaction on inner wheel decreases and becomes zero. So it leaves the ground first.

21. (b)

22. (a) $\frac{a_R}{a_r} = \frac{\omega_R^2 \times R}{\omega_r^2 \times r} = \frac{T_r^2}{T_R^2} \times \frac{R}{r} = \frac{R}{r}$ [As $T_r = T_R$]

23. (c) $\omega_{min} = \frac{2\pi}{60}\frac{Rad}{min}$ and $\omega_{hr} = \frac{2\pi}{12 \times 60}\frac{Rad}{min}$

$\therefore \frac{\omega_{min}}{\omega_{hr}} = \frac{2\pi/60}{2\pi/12 \times 60}$

24. (d) The particle performing circular motion flies off tangentially.

25. (a) The angle of banking, $\tan\theta = \frac{v^2}{rg}$

$\Rightarrow \tan 12° = \frac{(150)^2}{r \times 10} \Rightarrow r = 10.6 \times 10^3\, m = 10.6\, km$

26. (c) K.E. $= \frac{1}{2}mv^2$. Which is scalar, so it remains constant.

27. (b) $v = 72\, km/hour = 20\, m/\sec$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20 \times 20}{20 \times 10}\right) = \tan^{-1}(2)$

28. (a)

29. (d) $120\, rev/\min = 120 \times \frac{2\pi}{60}\, rad/\sec = 4\pi\, rad/\sec$

30. (c) In uniform circular motion, acceleration causes due to change in direction and is directed radially towards centre.

31. (b) Reaction on inner wheel $R_1 = \frac{1}{2}M\left[g - \frac{v^2 h}{ra}\right]$

Reaction on outer wheel $R_2 = \frac{1}{2}M\left[g + \frac{v^2 h}{ra}\right]$

where, r = radius of circular path, $2a$ = distance between two wheels and h = height of centre of gravity of car.

32. (d) Maximum tension $= m\omega^2 r = m \times 4\pi^2 \times n^2 \times r$

By substituting the values we get $T_{max} = 87.64\, N$

33. (d) $\frac{v^2}{rg} = \frac{h}{l} \Rightarrow v = \sqrt{\frac{rgh}{l}} = \sqrt{\frac{50 \times 1.5 \times 9.8}{10}} = 8.57\, m/s$

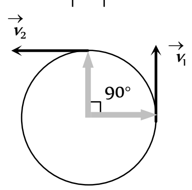
34. (b) $a = \omega^2 r = 4\pi^2 n^2 r = 4\pi^2 \times 1^2 \times 20 \times 10^3$

$\therefore$ a $= 8 \times 10^5\ m/\sec^2$

35. (c)

36. (d) In 15 second's hand rotate through 90°.

Change in velocity $|\vec{\Delta v}| = 2v\sin(\theta/2)$



$= 2(r\omega)\sin(90°/2) = 2 \times 1 \times \frac{2\pi}{T} \times \frac{1}{\sqrt{2}}$

$= \frac{4\pi}{60\sqrt{2}} = \frac{\pi\sqrt{2}}{30}\frac{cm}{sec}$ [As T = 60 sec]

37. (c) Since $n = 2$, $\omega = 2\pi \times 2 = 4\pi \, rad/s^2$

So acceleration $= \omega^2 r = (4\pi)^2 \times \frac{25}{100} m/s^2 = 4\pi^2$

38. (b) $\omega^2 r = 4\pi^2 n^2 r = 4\pi^2 \left(\frac{1200}{60}\right)^3 \times 30 = 4740 \, m/s^2$

39. (a)

40. (c) Particles of cream are lighter so they get deposited near the centre of circular path.

41. (d) Radial force $= \frac{mv^2}{r} = \frac{m}{r}\left(\frac{p}{m}\right)^2 = \frac{p^2}{mr}$ [As $p = mv$]

42. (b) $\frac{mv^2}{r} \propto \frac{K}{r} \Rightarrow v \propto r^o$

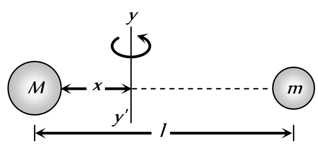
i.e. speed of the particle is independent of r.

43. (b) If the both mass are revolving about the axis yy' and tension in both the threads are equal then

$M\omega^2 x = m\omega^2(l - x)$

$\Rightarrow Mx = m(l - x)$

$\Rightarrow x = \frac{ml}{M + m}$



44. (b) $\tan\theta = \frac{v^2}{rg} = \frac{400}{20 \times 9.8} \Rightarrow \theta = 63.9°$

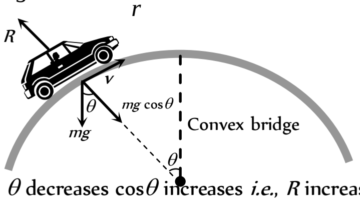
45. (d) In complete revolution change in velocity becomes zero so average acceleration will be zero.

46. (a) We know that $\tan\theta = \frac{v^2}{Rg}$ and $\tan\theta = \frac{h}{b}$

Hence $\frac{h}{b} = \frac{v^2}{Rg} \Rightarrow h = \frac{v^2 b}{Rg}$

47. (b)

48. (a) $R = mg\cos\theta - \frac{mv^2}{r}$



when θ decreases $\cos\theta$ increases *i.e.*, R increases.

49. (d) Tension in the string $T = m\omega^2 r = 4\pi^2 n^2 mr$

$\therefore T \propto n^2 \Rightarrow \frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow n_2 = 5\sqrt{\frac{2T}{T}} = 7 \, rpm$

50. (b)

51. (a) $T = m\omega^2 r \Rightarrow 10 = 0.25 \times \omega^2 \times 0.1 \Rightarrow \omega = 20 \, rad/s$

52. (c) $v = 36\frac{km}{h} = 10\frac{m}{s} \therefore F = \frac{mv^2}{r} = \frac{500 \times 100}{50} = 1000 N.$

53. (a) $T = \frac{mv^2}{r} \Rightarrow 25 = \frac{0.25 \times v^2}{1.96} \Rightarrow v = 14 \, m/s$

54. (b) Centripetal force $= mr\omega^2 = 5 \times 1 \times (2)^2 = 20 \, N$

55. (a) $\frac{mv^2}{r} = \frac{k}{r^2} \Rightarrow mv^2 = \frac{k}{r} \therefore$ K.E.$= \frac{1}{2}mv^2 = \frac{k}{2r}$

P.E. $= \int F\,dr = \int \frac{k}{r^2}dr = -\frac{k}{r}$

$\therefore$ Total energy = K.E. + P.E. $= \frac{k}{2r} - \frac{k}{r} = -\frac{k}{2r}$

56. (d) Maximum tension $= \frac{mv^2}{r} = 16N$

$\Rightarrow \frac{16 \times v^2}{144} = 16 \Rightarrow v = 12 \, m/s$

57. (a) The maximum velocity for a banked road with friction,

$v^2 = gr\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$

$\Rightarrow v^2 = 9.8 \times 1000 \times \left(\frac{0.5 + 1}{1 - 0.5 \times 1}\right) \Rightarrow v = 172 \, m/s$

58. (d) $v = r\omega = \frac{r \times 2\pi}{T} = \frac{0.06 \times 2\pi}{60} = 6.28 \, mm/s$

Magnitude of change in velocity $= |\vec{v}_2 - \vec{v}_1|$

$= \sqrt{v_1^2 + v_2^2} = 8.88 \, mm/s$ $(\text{As } v_1 = v_2 = 6.28 mm/s)$

59. (a) Work done by centripetal force in uniform circular motion is always equal to zero.

60. (b) $v = r\omega = 20 \times 10 \, cm/s = 2 \, m/s$

61. (d) $v_{max} = \sqrt{\mu rg} = \sqrt{0.2 \times 100 \times 9.8} = 14 m/s$

62. (d) $F = mg - \frac{mv^2}{r}$

63. (a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} rad/s$

64. (b) $\omega = 2\pi n = \frac{2\pi \times 100}{60} = 10.47 \, rad/s$

65. (d) Work done in circular motion is always zero.

66. (d) In complete revolution total displacement is zero so average velocity is zero

67. (c) $v_{max} = \sqrt{\mu rg} = \sqrt{0.75 \times 60 \times 9.8} = 21 m/s$

68. (a) Distance covered in 'n' revolution $= n \, 2\pi r = n\pi D$

$\Rightarrow 2000\pi D = 9500$ [As n = 2000, distance = 9500 m]

$\Rightarrow D = \frac{9500}{2000 \times \pi} = 1.5 m$

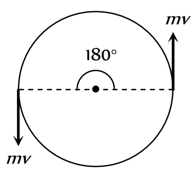
69. (c) Centripetal acceleration $= 4\pi^2 n^2 r = 4\pi^2 \times (1)^2 \times 0.4 = 1.6\pi^2$

70. (a)

71. (b) Due to centrifugal force.

72. (d) As momentum is vector quantity

$\therefore$ change in momentum



$\Delta P = 2mv\sin(\theta/2)$

$= 2mv\sin(90) = 2mv$

But kinetic energy remains always constant so change in kinetic energy is zero.

73. (a) $\omega = \frac{v}{r} = \frac{100}{100} = 1\,rad/s$

74. (c) $\alpha = \frac{d\omega}{dt} = 0$ (As ω = constant)

75. (b) $\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -4 & 1\\ 5 & -6 & 6\end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

76. (a) $a = 4\pi^2n^2r = 4\pi^2\left(\frac{1}{2}\right)^2 \times 50 = 493\,cm/s^2$

77. (c) Maximum force of friction = centripetal force

$\frac{mv^2}{r} = \frac{100\times(9)^2}{30} = 270\,N$

78. (a) $v = \sqrt{\mu rg} = \sqrt{0.4\times30\times9.8} = 10.84\,m/s$

79. (b) $v = r\omega = 0.5\times70 = 35\,m/s$

80. (a) $2\pi r = 34.3 \Rightarrow r = \frac{34.3}{2\pi}$ and $v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$

Angle of binding $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = 45°$

81. (c)

82. (a) $T = m\omega^2r \Rightarrow \omega \propto \sqrt{T}$ $\therefore \frac{\omega_2}{\omega_1} = \sqrt{\frac{1}{4}} \Rightarrow \omega_2 = \frac{\omega_1}{2} = 5\,rpm$

83. (d) $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left[\frac{(14\sqrt{3})^2}{20\sqrt{3}\times9.8}\right] = \tan^{-1}[\sqrt{3}] = 60°$

84. (c) Centripetal acceleration $= 4\pi^2n^2r = 4\pi^2\left(\frac{1}{2}\right)^2\times4 = 4\pi^2$

85. (b) Centripetal force = breaking force

$\Rightarrow m\omega^2r =$ breaking stress × cross sectional area

$\Rightarrow m\omega^2r = p\times A \Rightarrow \omega = \sqrt{\frac{p\times A}{mr}} = \sqrt{\frac{4.8\times10^7\times10^{-6}}{10\times0.3}}$

$\therefore\ \omega = 4\,rad/\sec$

86. (a) Because velocity is always tangential and centripetal acceleration is radial.

87. (c) T = tension, W = weight and F = centrifugal force.

88. (c) $\mu = \frac{v^2}{rg} = \frac{(4.9)^2}{4\times9.8} = 0.61$

89. (d) As body covers equal angle in equal time intervals. its angular velocity and hence magnitude of linear velocity is constant.

90. (b) $\omega = \frac{v}{r} = \frac{10}{100} = 0.1\,rad/s$

91. (a) $F = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{rF}{m}}$

92. (d) Electrostatic force provides necessary centripetal force for circular motion of electron.

93. (a) Acceleration $= \omega^2r = \frac{v^2}{r} = \omega v = \frac{2\pi}{T}v$

94. (b) $v = \sqrt{\mu rg} = \sqrt{0.6\times150\times10} = 30\,m/s$

95. (b)

96. (c) $F = \frac{mv^2}{r} \Rightarrow F \propto v^2$ *i.e.* force will become 4 times.

97. (d) $v = \sqrt{\mu rg} = \sqrt{0.25\times40\times10} = 10\,m/s$

98. (d) Time period = 40 *sec*

No. of revolution $= \frac{\text{Total time}}{\text{Time period}} = \frac{140\,\text{sec}}{40\,\text{sec}} = 3.5\,Rev$.

So, distance $= 3.5\times2\pi R = 3.5\times2\pi\times10 = 220\,m$.

99. (a) $m\,4\pi^2n^2r = 4\times10^{-13} \Rightarrow n = 0.08\times10^8\,cycles/\sec$.

100. (b) Momentum changes by $2mv$ but kinetic energy remains same.

101. (c) $L = I\omega$. In U.C.M. ω=constant $\therefore$ L = constant.

102. (c) $\because\ W = FS\cos\theta\ \therefore\ \theta = 90°$

103. (b)

104. (c) In uniform circular motion tangential acceleration remains zero but magnitude of radial acceleration remains constant.

105. (d) The inclination of person from vertical is given by,

$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{50\times10} = \frac{1}{5}$ $\therefore\ \theta = \tan^{-1}(1/5)$

106. (d) The centripetal force, $F = \frac{mv^2}{r} \Rightarrow r = \frac{mv^2}{F}$

$\therefore\ r \propto v^2$ or $v \propto \sqrt{r}$ (If m and F are constant),

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{1}{2}}$

107. (b) As the speed is constant throughout the circular motion therefore its average speed is equal to instantaneous speed.

108. (a) Linear velocity,

$v = \omega r = 2\pi nr = 2\times3.14\times3\times0.1 = 1.88\,m/s$

Acceleration, $a = \omega^2r = (6\pi)^2\times0.1 = 35.5\,m/s^2$

Tension in string, $T = m\omega^2r = 1\times(6\pi)^2\times0.1 = 35.5\,N$

109. (a) $a = \frac{v^2}{r} = \frac{(400)^2}{160} = 10^3\,m/s^2 = 1\,km/s^2$

110. (b) $v_{max.} = \sqrt{\mu rg} = \sqrt{0.5\times40\times9.8} = 14\,m/s$

111. (b) $F = \frac{mv^2}{r} = \frac{500\times100}{50} = 10^3\,N$

112. (b) $F = m\left(\frac{4\pi^2}{T^2}\right)R$. If masses and time periods are same then $F \propto R \ \therefore \ F_1/F_2 = R_1/R_2$

113. (b) It is a vector quantity.

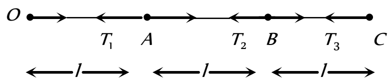
114. (a) $a = \frac{v^2}{r} = v\omega \Rightarrow a' = (2v)\left(\frac{\omega}{2}\right) = a$ *i.e.* remains constant.

115. (d) Tension in the string $T_0 = mR\omega_0^2$

In the second case $T = m(2R)(4\omega_0^2) = 8mR\omega_0^2 = 8T_0$

116. (b) Average velocity $= \frac{\text{Total displacement}}{\text{Total time}} = \frac{2m}{1s} = 2ms^{-1}$

117. (d) Let ω is the angular speed of revolution



$T_3 = m\omega^2 3l$

$T_2 - T_3 = m\omega^2 2l \Rightarrow T_2 = m\omega^2 5l$

$T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = m\omega^2 6l$

$T_3 : T_2 : T_1 = 3 : 5 : 6$

118. (b) $F = \frac{mv^2}{r}$. For same mass and same speed if radius is doubled then force should be halved.

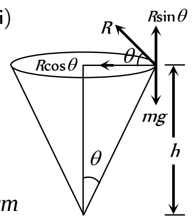
119. (c) $a = \frac{v^2}{r} = \omega^2 r = 4\pi^2 n^2 r = 4\pi^2\left(\frac{22}{44}\right)^2 \times 1 = \pi^2 \ m/s^2$

and its direction is always along the radius and towards the centre.

120. (d) The particle is moving in circular path

From the figure, $mg = R\sin\theta$ ...(i)

$\frac{mv^2}{r} = R\cos\theta$ ...(ii)



From equation (i) and (ii) we get

$\tan\theta = \frac{rg}{v^2}$ but $\tan\theta = \frac{r}{h}$

$\therefore h = \frac{v^2}{g} = \frac{(0.5)^2}{10} = 0.025m = 2.5\,cm$

121. (a) Angular velocity $= \frac{2\pi}{T} = \frac{2\pi}{24} rad/hr = \frac{2\pi}{86400} rad/s$

122. (d) $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.1047\ rad/s$

and $v = \omega r = 0.1047 \times 3 \times 10^{-2} = 0.00314\ m/s$

Non-uniform Circular Motion

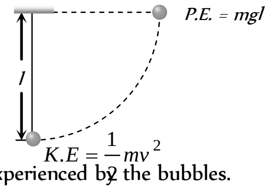
1. (d) Minimum speed at the highest point of vertical circular path $v = \sqrt{gR}$

2. (d) At highest point $\frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$

3. (d) Kinetic energy given to a sphere at lowest point = potential energy at the height of suspension

$\Rightarrow \frac{1}{2}mv^2 = mgl$

$\therefore v = \sqrt{2gl}$



4. (c) Due to less centrifugal force experienced by the bubbles.

5. (a) Critical velocity at highest point $= \sqrt{gR} = \sqrt{10 \times 1.6} = 4m/s$

6. (c) Using relation $\theta = \omega_0 t + \frac{1}{2}at^2$

$\theta_1 = \frac{1}{2}(\alpha)(2)^2 = 2\alpha$ ...(i) (As $\omega_0 = 0, t = 2$ sec)

Now using same equation for $t = 4$ sec, $\omega_0 = 0$

$\theta_1 + \theta_2 = \frac{1}{2}\alpha(4)^2 = 8\alpha$ ...(ii)

From (i) and (ii), $\theta_1 = 2\alpha$ and $\theta_2 = 6\alpha \ \therefore \ \frac{\theta_2}{\theta_1} = 3$

7. (a) $mg = 1 \times 10 = 10N$, $\frac{mv^2}{r} = \frac{1 \times (4)^2}{1} = 16$

Tension at the top of circle $= \frac{mv^2}{r} - mg = 6N$

Tension at the bottom of circle $= \frac{mv^2}{r} + mg = 26N$

8. (d) For critical condition at the highest point $\omega = \sqrt{g/R}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{R/g} = 2 \times 3.14\sqrt{4/9.8} = 4$ sec.

9. (b) $mg = 20N$ and $\frac{mv^2}{r} = \frac{2 \times (4)^2}{1} = 32N$

It is clear that 52 N tension will be at the bottom of the circle.

Because we know that $T_{\text{Bottom}} = mg + \frac{mv^2}{r}$

10. (b) $h = \frac{5}{2}R = \frac{5}{2}\left(\frac{D}{2}\right) = \frac{5D}{4}$

11. (b) Net acceleration in nonuniform circular motion,

$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left(\frac{900}{500}\right)^2} = 2.7\ m/s^2$

a_t = tangential acceleration

a_c = centripetal acceleration $= \frac{v^2}{r}$

12. (b) $T = mg + \frac{mv^2}{l} = mg + 2mg = 3mg$

where $v = \sqrt{2gl}$ from $\frac{1}{2}mv^2 = mgl$

13. (a) $T_{max} = m\omega_{max}^2 r + mg \Rightarrow \frac{T_{max}}{m} = \omega^2 r + g$

$\Rightarrow \frac{30}{0.5} - 10 = \omega^2{}_{max} r \Rightarrow \omega_{max} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5\,rad/s$

14. (b)

15. (b) Because here tension is maximum.

16. (a) Max. tension that string can bear = 3.7 *kgwt* = 37*N*

Tension at lowest point of vertical loop $= mg + m\omega^2 r$

=

$0.5 \times 10 + 0.5 \times \omega^2 \times 4 = 5 + 2\omega^2$

$\therefore$ 37 = 5 + 2ω $\Rightarrow$ ω = 4 rad/s.

17. (c)

18. (c) $\omega = \frac{d\theta}{dt} = \frac{d}{dt}(2t^3 + 0.5) = 6t^2$

at t =2 *s*, $\omega = 6 \times (2)^2 = 24 rad/s$

19. (a) When body is released from the position *p* (inclined at angle θ from vertical) then velocity at mean position

$v = \sqrt{2gl(1 - \cos\theta)}$

$\therefore$ Tension at the lowest point = $mg + \frac{mv^2}{l}$

$= mg + \frac{m}{l}[2gl(1 - \cos 60)] = mg + mg = 2mg$

20. (a)

21. (c) Tension = Centrifugal force + weight $= \frac{mv^2}{r} + mg$

22. (a) $v_{min} = \sqrt{5\,gr} = 17.7\,m/\sec$

23. (d)

24. (c) $v = \sqrt{2gl(1 - \cos\theta)} = \sqrt{2 \times 9.8 \times 2(1 - \cos 60°)} = 4.43\,m/s$

25. (b) Increment in angular velocity $\omega = 2\pi(n_2 - n_1)$

$\omega = 2\pi(1200 - 600)\frac{rad}{min} = \frac{2\pi \times 600}{60}\frac{rad}{s} = 20\pi\frac{rad}{s}$

26. (d) $\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.8}{0.2}} = 7\,rad/s$

27. (a)

28. (d) In non-uniform circular motion particle possess both centripetal as well as tangential acceleration.

29. (c)

30. (b) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2\,m/s$

31. (d) $T = mg + m\omega^2 r = m\{g + 4\pi^2 n^2 r\}$

$= m\left\{g + \left(4\pi^2\left(\frac{n}{60}\right)^2 r\right)\right\} = m\left\{g + \left(\frac{\pi^2 n^2 r}{900}\right)\right\}$

32. (d) $h = \frac{5}{2}r \Rightarrow r = \frac{2}{5} \times h = \frac{2}{5} \times 5 = 2\,metre$

33. (d) In the given condition friction provides the required centripetal force and that is constant. *i.e.* $m\omega r$=constant

$\Rightarrow r \propto \frac{1}{\omega^2} \therefore r_2 = r_1\left(\frac{\omega_1}{\omega_2}\right)^2 = 9\left(\frac{1}{3}\right)^2 = 1\,cm$

34. (b) By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha(2\pi n) \Rightarrow \alpha = \frac{3}{4}\frac{\omega_0^2}{4\pi \times 36}$, $(n = 36)$ ...(i)

Now let fan completes total *n′* revolution from the starting to come to rest

$0 = \omega_0^2 - 2\alpha(2\pi n') \Rightarrow n' = \frac{\omega_0^2}{4\alpha\pi}$

substituting the value of α from equation (i)

$n' = \frac{\omega_0^2}{4\pi}\frac{4 \times 4\pi \times 36}{3\omega_0^2} = 48$ revolution

Number of rotation = 48 – 36 = 12

35. (b) $v = \sqrt{3gr}$ and $a = \frac{v^2}{r} = \frac{3gr}{r} = 3g$

36. (d) Tension at mean position, $mg + \frac{mv^2}{r} = 3mg$

$v = \sqrt{2gl}$ ...(i)

and if the body displaces by angle θ with the vertical then

$v = \sqrt{2gl(1 - \cos\theta)}$ ...(ii)

Comparing (i) and (ii), $\cos\theta = 0 \Rightarrow \theta = 90°$

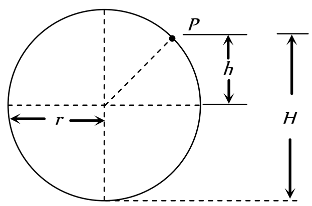
37. (c) Tension, $T = \frac{mv^2}{r} + mg\cos\theta$

For, $\theta = 30°, T_1 = \frac{mv^2}{r} + mg\cos 30°$

$\theta = 60°, T_2 = \frac{mv^2}{r} + mg\cos 60° \therefore T_1 > T_2$

38. (c) As we know for hemisphere the particle will leave the sphere at height $h = 2r/3$

$h = \frac{2}{3} \times 21 = 14\,m$



but from the bottom

$H = h + r = 14 + 21 = 35\,metre$

39. (c) $x = \alpha t^3$ and $y = \beta t^3$ (given)

$v_x = \frac{dx}{dt} = 3\alpha t^2$ and $v_y = \frac{dy}{dt} = 3\beta t^2$

Resultant velocity $= v = \sqrt{v_x^2 + v_y^2} = 3t^2\sqrt{\alpha^2 + \beta^2}$

40. (b)

41. (d) Tension at the top of the circle, $T = m\omega^2 r - mg$

$T = 0.4 \times 4\pi^2 n^2 \times 2 - 0.4 \times 9.8 = 115.86\,N$

42. (c) Minimum angular velocity $\omega_{min} = \sqrt{g/R}$

$\therefore T_{max} = \frac{2\pi}{\omega_{min}} = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{2}{10}} = 2\sqrt{2} \cong 3s$

43. (a) $|\overrightarrow{\Delta v}| = 2v\sin(\theta/2) = 2v\sin\left(\frac{90}{2}\right) = 2v\sin 45 = v\sqrt{2}$

44. (a) In this problem it is assumed that particle although moving in a vertical loop but its speed remain constant.

Tension at lowest point $T_{max} = \frac{mv^2}{r} + mg$

Tension at highest point $T_{min} = \frac{mv^2}{r} - mg$

$\frac{T_{max}}{T_{min}} = \frac{\frac{mv^2}{r} + mg}{\frac{mv^2}{r} - mg} = \frac{5}{3}$

by solving we get, $v = \sqrt{4gr} = \sqrt{4 \times 9.8 \times 2.5} = \sqrt{98}\,m/s$

45. (d) There is no relation between centripetal and tangential acceleration. Centripetal acceleration is must for circular motion but tangential acceleration may be zero.

46. (d) Angular momentum is a axial vector. It is directed always in a fix direction (perpendicular to the plane of rotation either outward or inward), if the sense of rotation remain same.

47. (a) Difference in kinetic energy = $2mgr = 2 \times 1 \times 10 \times 1 = 20J$

48. (d) Angular acceleration = $\frac{d^2\theta}{dt^2} = 2\theta$

Horizontal Projectile Motion

1. (b) $R_{max} = \frac{u^2}{g} = 16 \times 10^3 \Rightarrow u = 400\,m/s$

2. (c) Due to constant velocity along horizontal and vertical downward force of gravity stone will hit the ground following parabolic path.

3. (b) Because the vertical components of velocities of both the bullets are same and equal to zero and $t = \sqrt{\frac{2h}{g}}$.

4. (c) The pilot will see the ball falling in straight line because the reference frame is moving with the same horizontal velocity but the observer at rest will see the ball falling in parabolic path.

5. (b) Due to air resistance, it's horizontal velocity will decrease so it will fall behind the aeroplane.

6. (c) Because horizontal velocity is same for coin and the observer. So relative horizontal displacement will be zero.

7. (c) Horizontal displacement of the bomb

AB = Horizontal velocity × time available

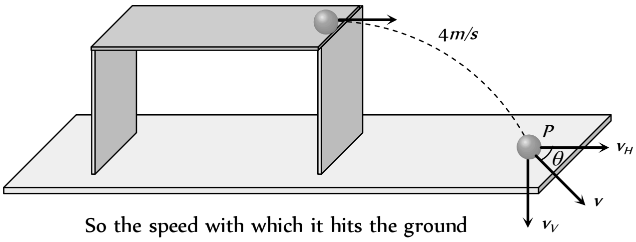
$AB = u \times \sqrt{\frac{2h}{g}} = 600 \times \frac{5}{18} \times \sqrt{\frac{2 \times 1960}{9.8}} = 3.33\,Km.$

8. (a,c) Vertical component of velocity of ball at point P

$v_V = 0 + gt = 10 \times 0.4 = 4\,m/s$

Horizontal component of velocity = initial velocity

$\Rightarrow v_H = 4\,m/s$



So the speed with which it hits the ground

$v = \sqrt{v_H^2 + v_V^2} = 4\sqrt{2}\,m/s$

and $\tan\theta = \frac{v_V}{v_H} = \frac{4}{4} = 1 \Rightarrow \theta = 45°$

It means the ball hits the ground at an angle of 45° to the horizontal.

Height of the table $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.4)^2 = 0.8\,m$

Horizontal distance travelled by the ball from the edge of table $h = ut = 4 \times 0.4 = 1.6\,m$

9. (b) $S = u \times \sqrt{\frac{2h}{g}} = 100 \times \sqrt{\frac{2 \times 490}{9.8}} = 1000\,m = 1\,km$

10. (c) $S = u \times \sqrt{\frac{2h}{g}} \Rightarrow 10 = u\sqrt{2 \times \frac{5}{10}} \Rightarrow u = 10\,m/s$

11. (d) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 396.9}{9.8}} \simeq 9$ sec and $u = 720\,km/hr = 200\,m/s$

$\therefore R = u \times t = 200 \times 9 = 1800\,m$

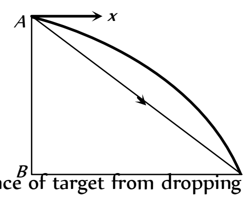
12. (a) For both cases $t = \sqrt{\frac{2h}{g}}$ = constant.

Because vertical downward component of velocity will be zero for both the particles.

13. (c)

14. (a) The horizontal distance covered by bomb,

$BC = v_H \times \sqrt{\frac{2h}{g}} = 150\sqrt{\frac{2 \times 80}{10}} = 660m$

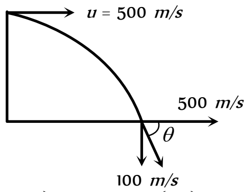


$\therefore$ The distance of target from dropping point of bomb,

$AC = \sqrt{AB^2 + BC^2} = \sqrt{(80)^2 + (600)^2} = 605.3\,m$

15. (a) Horizontal component of velocity $v = 500\,m/s$

and vertical components of velocity while striking the ground.



$v_y = 0 + 10\times 10 = 100\, m/s$

$\therefore$ Angle with which it strikes the ground.

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) = \tan^{-1}\left(\frac{1}{5}\right)$$

16. (b) Area in which bullet will spread $= \pi r^2$

For maximum area, $r = R_{max} = \frac{v^2}{g}$ [when $\theta = 45°$]

Maximum area $\pi R_{max}^2 = \pi\left(\frac{v^2}{g}\right)^2 = \frac{\pi v^4}{g^2}$

Oblique Projectile Motion

1. (d) $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore R \propto u^2$. If initial velocity be doubled then range will become four times.

2. (c) $H = \frac{u^2 \sin^2\theta}{2g}$ $\therefore H \propto u^2$. If initial velocity be doubled then maximum height reached by the projectile will quadrupled.

3. (a) An external force by gravity is present throughout the motion so momentum will not be conserved.

4. (a) Range $= \frac{u^2 \sin 2\theta}{g}$; when $\theta = 90°$, $R = 0$ *i.e.* the body will fall at the point of projection after completing one dimensional motion under gravity.

5. (c) $R = 4H\cot\theta$.

When $R = H$ then $\cot\theta = 1/4 \Rightarrow \theta = \tan^{-1}(4)$

6. (c) Because there is no accelerating or retarding force available in horizontal motion.

7. (a) Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means angle between $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

8. (d) $R = 4H\cot\theta$ if $\theta = 45°$ then $R = 4H\cot(45°) = 4H$

9. (c) $v_y = \frac{dy}{dt} = 8 - 10t$, $v_x = \frac{dx}{dt} = 6$

at the time of projection *i.e.* $v_y = \frac{dy}{dt} = 8$ and $v_x = 6$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10\ m/s$

10. (b) The angle of projection is given by $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

11. (a) $a_x = \frac{d}{dt}(v_x) = 0$, $a_y = \frac{d}{dt}(v_y) = -10\ m/s^2$

$\therefore$ Net acceleration $a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + 10^2} = 10\, m/s$

12. (b) $R_{15°} = \frac{u^2 \sin(2\times 15°)}{g} = \frac{u^2}{2g} = 1.5\, km$

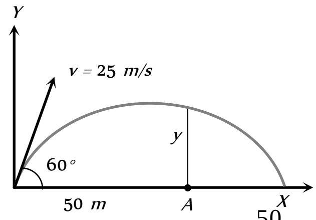
$R_{45°} = \frac{u^2 \sin(2\times 45°)}{g} = \frac{u^2}{g} = 1.5\times 2 = 3\, km$

13. (a) Horizontal component of velocity

$v_x = 25\cos 60° = 12.5\, m/s$

Vertical component of velocity

$v_y = 25\sin 60° = 12.5\sqrt{3}\ m/s$



Time to cover 50 m distance $t = \frac{50}{12.5} = 4$ sec

The vertical height y is given by

$$y = v_y t - \frac{1}{2}gt^2 = 12.5\sqrt{3}\times 4 - \frac{1}{2}\times 9.8\times 16 = 8.2\, m$$

14. (a) For vertical upward motion $h = ut - \frac{1}{2}gt^2$

$$5 = (25\sin\theta)\times 2 - \frac{1}{2}\times 10\times (2)^2$$

$\Rightarrow 25 = 50\sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

15. (c) For angle $(45° - \theta)$, $R = \frac{u^2 \sin(90° - 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

For angle $(45° + \theta)$, $R = \frac{u^2 \sin(90° + 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

16. (b) Range is given by $R = \frac{u^2 \sin 2\theta}{g}$

On moon $g_m = \frac{g}{6}$. Hence $R_m = 6R$

17. (c) For greatest height $\theta = 90°$

$H_{max} = \frac{u^2 \sin^2(90°)}{2g} = \frac{u^2}{2g} = h$ (given)

$R_{max} = \frac{u^2 \sin^2 2(45°)}{g} = \frac{u^2}{g} = 2h$

18. (c) $R = 4H\cot\theta$, if $R = 4H$ then $\cot\theta = 1 \Rightarrow \theta = 45°$

19. (b) $E' = E\cos^2\theta = E\cos^2(45°) = \frac{E}{2}$

20. (b)

21. (b)

22. (d) Acceleration through out the projectile motion remains constant and equal to g.

23. (c)

24. (c) Time of flight $= \frac{2u\sin\theta}{g} = \frac{2\times 50\times \sin 30}{10} = 5s$

25. (b) Change in momentum $= 2mu\sin\theta$

$= 2 \times 0.5 \times 98 \times \sin 30 = 45 N\text{-}s$

26. (d) $R = 4H\cot\theta$, if $R = 3H$ then $\cot\theta = \frac{3}{4} \Rightarrow \theta = 53°8'$

27. (c) Became vertical downward displacement of both (barrel and bullet) will be equal.

28. (b) As $H = \frac{u^2 \sin^2\theta}{2g}$ $\therefore \frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin\theta_2} = \frac{\sin^2 30°}{\sin^2 60} = \frac{1/4}{3/4} = \frac{1}{3}$

29. (d) $R = \frac{v^2 \sin 2\theta}{g} \Rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{v^2}\right)$

30. (a) $T = \frac{2u\sin\theta}{g} = 10 \sec \Rightarrow u\sin\theta = 50\ m/s$

$\therefore H = \frac{u^2\sin^2\theta}{2g} = \frac{(u\sin\theta)^2}{2g} = \frac{50 \times 50}{2 \times 10} = 125m$

31. (b) For complementary angles range will be equal.

32. (b) $R = \frac{u^2 \sin 2\theta}{g} = \frac{(500)^2 \times \sin 30°}{10} = 12.5 \times 10^3\ m$

33. (a) $T = \frac{2u\sin\theta}{g} \Rightarrow u = \frac{T \times g}{2\sin\theta} = \frac{2 \times 9.8}{2 \times \sin 30} = 19.6\ m/s$

34. (c) $R = \frac{u^2 \sin 2\theta}{g} = R \propto u^2$. So if the speed of projection doubled, the range will becomes four times, *i.e.*, $4 \times 50 = 200\ m$

35. (c) Range will be equal for complementary angles.

36. (a) When the angle of projection is very far from 45° then range will be minimum.

37. (a) $H = \frac{u^2\sin^2\theta}{2g}$ and $T = \frac{2u\sin\theta}{g}$

So $\frac{H}{T^2} = \frac{u^2\sin^2\theta / 2g}{4u^2\sin^2\theta / g^2} = \frac{g}{8} = \frac{5}{4}$

38. (a) $H_1 = \frac{u^2\sin^2\theta}{2g}$ and $H_2 = \frac{u^2\sin^2(90-\theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$

$H_1H_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g} = \frac{(u^2\sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$

$\therefore R = 4\sqrt{H_1H_2}$

39. (d) Standard equation of projectile motion

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Comparing with given equation

$A = \tan\theta$ and $B = \frac{g}{2u^2\cos^2\theta}$

So $\frac{A}{B} = \frac{\tan\theta \times 2u^2\cos^2\theta}{g} = 40$

(As $\theta = 45°, u = 20\ m/s, g = 10\ m/s^2$)

40. (b) Range $= \frac{u^2\sin 2\theta}{g}$. It is clear that range is proportional to the direction (angle) and the initial speed.

41. (c) $\frac{2u\sin\theta}{g} = 2\sec \Rightarrow u\sin\theta = 10$

$\therefore H = \frac{u^2\sin^2\theta}{2g} = \frac{100}{2g} = 5m$

42. (b) Only horizontal component of velocity $(u\cos\theta)$.

43. (a) For complementary angles range is same.

44. (b) $T = \frac{2u\sin\theta}{g} = \frac{2 \times 9.8 \times \sin 30}{9.8} = 1\,s$

45. (a) $x = 36t \therefore v_x = \frac{dx}{dt} = 36\,m/s$

$y = 48t - 4.9t^2 \therefore v_y = 48 - 9.8t$

at $t = 0$ $v_x = 36$ and $v_y = 48\,m/s$

So, angle of projection $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

Or $\theta = \sin^{-1}(4/5)$

46. (b) For same range angle of projection should be θ and $90-\theta$

So, time of flights $t_1 = \frac{2u\sin\theta}{g}$ and

$t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$

By multiplying $= t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$

$t_1t_2 = \frac{2}{g}\frac{(u^2\sin 2\theta)}{g} = \frac{2R}{g} \Rightarrow t_1t_2 \propto R$

47. (c) Instantaneous velocity of rising mass after t sec will be $v_t = \sqrt{v_x^2 + v_y^2}$

where $v_x = v\cos\theta =$ Horizontal component of velocity

$v_y = v\sin\theta - gt =$ Vertical component of velocity

$v_t = \sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$

$v_t = \sqrt{v^2 + g^2t^2 - 2v\sin\theta\, gt}$

48. (d) Maximum range $= \frac{u^2}{g} = 100\,m$

Maximum height $= \frac{u^2}{2g} = \frac{100}{2} = 50\,m$

49. (c) $R_{max} = \frac{u^2}{g} = 100 \Rightarrow u = 10\sqrt{10} = 32\,m/s$

50. (c) Since horizontal component of velocity is constant, hence momentum is constant.

51. (a) Time of flight $= \frac{2u\sin\theta}{g} = \frac{2u_y}{g} = \frac{2 \times u_{\text{vertical}}}{g}$

52. (a) Person will catch the ball if its velocity will be equal to horizontal component of velocity of the ball.

$\frac{v_0}{2} = v_0\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

53. (b) $H = \frac{u^2\sin^2\theta}{2g}$ and $T = \frac{2u\sin\theta}{g} \Rightarrow T^2 = \frac{4u^2\sin^2\theta}{g^2}$

$\therefore \frac{T^2}{H} = \frac{8}{g} \Rightarrow T = \sqrt{\frac{8H}{g}} = 2\sqrt{\frac{2H}{g}}$

54. (d) $R = 4H\cot\theta$, if $R = 4\sqrt{3}H$ then $\cot\theta = \sqrt{3} \Rightarrow \theta = 30°$

55. (c) The vertical component of velocity of projection $= -50\sin 30° = -25\,m/s$

If t be the time taken to reach the ground,

$h = ut + \frac{1}{2}gt^2 \Rightarrow 70 = -25t + \frac{1}{2}\times 10t^2$

$\Rightarrow 70 = -25t + 5t^2 \Rightarrow t^2 - 5t - 14 = 0 \Rightarrow t = -2s$ and $7s$

Since, $t = -2\ s$ is not valid $\therefore t = 7\ s$

56. (c) $H = \frac{u^2\sin^2\theta}{2g}$

According to problem $\frac{u_1^{\,2}\sin^2 45°}{2g} = \frac{u_2^{\,2}\sin^2 60°}{2g}$

$\Rightarrow \frac{u_1^{\,2}}{u_2^{\,2}} = \frac{\sin^2 60°}{\sin^2 45°} \Rightarrow \frac{u_1}{u_2} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \sqrt{\frac{3}{2}}.$

57. (c)

58. (d) $R = 4H\cot\theta$, if $\theta = 45°$ then $R = 4H \Rightarrow \frac{R}{H} = \frac{4}{1}$

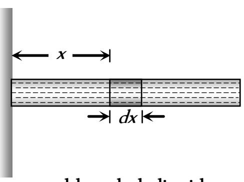
59. (b) $R_{max} = \frac{u^2}{g} = 400m$ (For $\theta = 45°$)

$H_{max} = \frac{u^2}{2g} = \frac{400}{2} = 200m$ (For $\theta = 90°$)

Critical Thinking Questions

1. (c,d) In the given condition, the particle undergoes uniform circular motion and for uniform circular motion the velocity and acceleration vector changes continuously but kinetic energy is constant for every point.

2. (a) $dM = \left(\frac{M}{L}\right)dx$



force on 'dM' mass is

$dF = (dM)\omega^2 x$

By integration we can get the force exerted by whole liquid

$\Rightarrow F = \int_0^L \frac{M}{L}\omega^2 x\,dx = \frac{1}{2}M\omega^2 L$

3. (b) According to given problem $\frac{1}{2}mv^2 = as^2 \Rightarrow v = s\sqrt{\frac{2a}{m}}$

So $a_R = \frac{v^2}{R} = \frac{2as^2}{mR}$ ...(i)

Further more as $a_t = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt} = v\frac{dv}{ds}$ ...(ii)

(By chain rule)

Which in light of equation (i) *i.e.* $v = s\sqrt{\frac{2a}{m}}$ yields

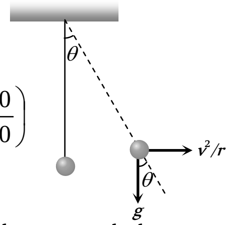
$a_t = \left[s\sqrt{\frac{2a}{m}}\right]\left[\sqrt{\frac{2a}{m}}\right] = \frac{2as}{m}$ ...(iii)

So that $a = \sqrt{a_R^2 + a_t^2} = \sqrt{\left[\frac{2as^2}{mR}\right]^2 + \left[\frac{2as}{m}\right]^2}$

Hence $a = \frac{2as}{m}\sqrt{1 + [s/R]^2}$

$\therefore F = ma = 2as\sqrt{1 + [s/R]^2}$

4. (c) $\tan\theta = \frac{v^2/r}{g} = \frac{v^2}{rg}$



$\therefore \theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{10\times 10}{10\times 10}\right)$

$\therefore \theta = \tan^{-1}(1) = 45°$

5. (b) Here the tangential acceleration also exits which requires power.

Given that $a_C = k^2rt^2$ and $a_C = \frac{v^2}{r}$ $\therefore \frac{v^2}{r} = k^2rt^2$

or $v^2 = k^2r^2t^2$ or $v = krt$

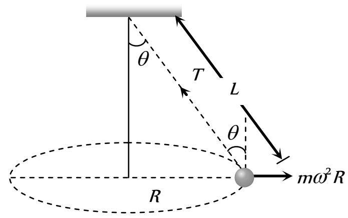
Tangential acceleration $a = \frac{dv}{dt} = kr$

Now force $F = m\times a = mkr$

So power $P = F\times v = mkr\times krt = mk^2r^2t$

6. (d) $T\sin\theta = M\omega^2 R$ ...(i)

$T\sin\theta = M\omega^2 L\sin\theta$ ...(ii)



From (i) and (ii)

$T = M\omega^2 L$

$= M\,4\pi^2 n^2 L$

$= M\,4\pi^2\left(\frac{2}{\pi}\right)^2 L$

$= 16\ ML$

7. (d) Since the maximum tension T_B in the string moving in the vertical circle is at the bottom and minimum tension T_T is at the top.

$\therefore T_B = \frac{mv_B^2}{L} + mg$ and $T_T = \frac{mv_T^2}{L} - mg$

$\therefore \frac{T_B}{T_T} = \frac{\frac{mv_B^2}{L} + mg}{\frac{mv_T^2}{L} - mg} = \frac{4}{1}$ or $\frac{v_B^2 + gL}{v_T^2 - gL} = \frac{4}{1}$

or $v_B^2 + gL = 4v_T^2 - 4gL$ but $v_B^2 = v_T^2 + 4gL$

$\therefore v_T^2 + 4gL + gL = 4v_T^2 - 4gL \Rightarrow 3v_T^2 = 9gL$

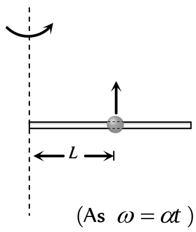
$\therefore\ v_T^2 = 3\times g\times L = 3\times 10\times\dfrac{10}{3}$ or $v_T = 10\,m/\sec$

8. (a) For particle P, motion between A and C will be an accelerated one while between C and B a retarded one. But in any case horizontal component of it's velocity will be greater than or equal to v on the other hand in case of particle Q, it is always equal to v. horizontal displacement of both the particles are equal, so $t_P < t_Q$.

9. (a) Let the bead starts slipping after time t

For critical condition

Frictional force provides the centripetal force



$m\omega^2 L = \mu R = \mu m\times a_t = \mu L m\alpha$

$\Rightarrow m(\alpha t)^2 L = \mu m L\alpha \Rightarrow t = \sqrt{\dfrac{\mu}{\alpha}}$ (As $\omega = \alpha t$)

10. (a) Normal reaction at the highest point

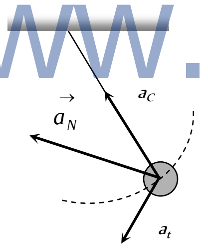
$$R = \frac{mv^2}{r} - mg$$

Reaction is inversely proportional to the radius of the curvature of path and radius is minimum for path depicted in (a).

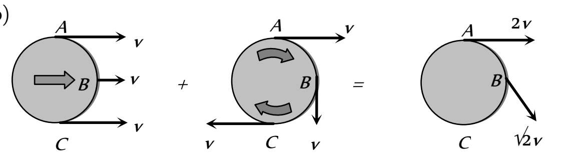
11. (c) a_c = centripetal acceleration

a_t = tangential acceleration

a_N = net acceleration = Resultant of a_c and a_t



12. (b)



Pure translation + Pure Rotation = Rolling without Slipping

13. (d) $\dfrac{1}{2}mu^2 - \dfrac{1}{2}mv^2 = mgL$

$\Rightarrow v = \sqrt{u^2 - 2gL}$

$|\vec{v}-\vec{u}| = \sqrt{u^2+v^2} = \sqrt{u^2+u^2-2gL} = \sqrt{2(u^2-gL)}$

14. (a) When driver applies brakes and the car covers distance x before coming to rest, under the effect of retarding force F

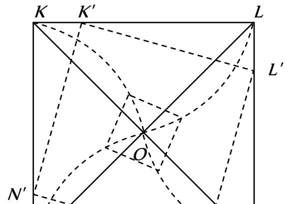
then $\dfrac{1}{2}mv^2 = Fx \Rightarrow x = \dfrac{mv^2}{2F}$

But when he takes turn then $\dfrac{mv^2}{r} = F \Rightarrow r = \dfrac{mv^2}{F}$

It is clear that $x = r/2$

i.e. by the same retarding force the car can be stopped in a less distance if the driver apply breaks. This retarding force is actually a friction force.

15. (a) It is obvious from considerations of symmetry that at any moment of time all of the persons will be at the corners of square whose side gradually decreases (see fig.) and so they will finally meet at the centre of the square O.



The speed of each person along the line joining his initial position and O will be $v\cos 45 = v/\sqrt{2}$.

As each person has displacement $d\cos 45 = d/\sqrt{2}$ to reach the centre, the four persons will meet at the centre of the square O after time.

$\therefore\ t = \dfrac{d/\sqrt{2}}{v/\sqrt{2}} = \dfrac{d}{v}$

16. (a,b) $x = a\cos(pt)$ and $y = b\sin(pt)$ (given)

$\therefore\ \cos pt = \dfrac{x}{a}$ and $\sin pt = \dfrac{y}{b}$

By squaring and adding

$\cos^2(pt)+\sin^2(pt) = \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2} = 1$

Hence path of the particle is ellipse.

Now differentiating x and y *w.r.t.* time

$v_x = \dfrac{dx}{dt} = \dfrac{d}{dt}(a\cos(pt)) = -ap\sin(pt)$

$v_y = \dfrac{dy}{dt} = \dfrac{d}{dt}(b\sin(pt)) = bp\cos(pt)$

$\therefore\ \vec{v} = v_x\hat{i} + v_y\hat{j} = -ap\sin(pt)\hat{i} + bp\cos(pt)\hat{j}$

Acceleration $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}[-ap\sin(pt)\hat{i} + bp\cos(pt)\hat{j}]$

$\vec{a} = -ap^2\cos(pt)\,\hat{i} - bp^2\sin(pt)\hat{j}$

Velocity at $t = \dfrac{\pi}{2p}$

$\vec{v} = -ap\sin p\left(\dfrac{\pi}{2p}\right)\hat{i} + bp\cos p\left(\dfrac{\pi}{2p}\right)\hat{j} = -ap\,\hat{i}$

Acceleration at $t = \dfrac{\pi}{2p}$

$\vec{a} = ap^2\cos p\left(\dfrac{\pi}{2p}\right)\hat{i} - bp^2\sin p\left(\dfrac{\pi}{2p}\right)\hat{j} = -bp^2\hat{j}$

As $\vec{v}\,.\,\vec{a} = 0$

Hence velocity and acceleration are perpendicular to each other at $t = \dfrac{\pi}{2p}$.

17. (b) $\Delta\vec{\upsilon} = \vec{\upsilon}_2 - \vec{\upsilon}_1 = \sqrt{\upsilon_1^2+\upsilon_2^2-2\upsilon_1\upsilon_2\cos 90^o} = \sqrt{5^2+5^2} = 5\sqrt{2}$

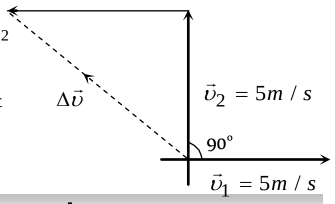
$-\vec{v}_1$

Average acceleration

$= \frac{\Delta v}{\Delta t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s ^{2}

Directed toward north-west

(As clear from the figure).



Graphical Questions

1. (d) $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x v_y}{g}$

$\therefore$ Range $\propto$ horizontal initial velocity (u)

In path 4 range is maximum so football possess maximum horizontal velocity in this path.

2. (a) If air resistance is taken into consideration then range and maximum height, both will decrease.

3. (b)

4. (c)

5. (d)

Assertion and Reason

1. (e) At the highest point, vertical component of velocity becomes zero so there will be only horizontal velocity and it is perpendicular to the acceleration due to gravity.

2. (a) $H = \frac{u^2 \sin^2 \theta}{2g}$ *i.e.* it is independent of mass of projectile.

3. (c) $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore R_{max} = \frac{u^2}{g}$ when $\theta = 45° \therefore R_{max} \propto u^2$

Height $H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow H_{max} = \frac{u^2}{2g}$ when $\theta = 90°$

It is clear that $H_{max} = \frac{R_{max}}{2}$

4. (c) Horizontal range depends upon angle of projection and it is same for complementary angles *i.e.* θ and $(90 - \theta)$.

5. (b) We know $R = 4H \cot\theta$

if $R = H$ then $\cot\theta = \left(\frac{1}{4}\right)$ or $\tan \theta = (4)$

and $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore R \propto \frac{u^2}{g}$

6. (d) $y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$

7. (d) If a body is projected from a place above the surface of earth, then for the maximum range, the angle of projection should be slightly less than 45^o.

8. (a) Both body will take same time to reach the earth because vertical downward component of velocity for both the bodies will be zero and time of descent $t = \sqrt{\frac{2h}{g}}$. Horizontal velocity has no effect on the vertical direction.

9. (c) $T \propto u$ and $R \propto u^2$

When velocity of projection of a body is made n times, then its time of flight becomes n times and range becomes n times.

10. (c) Range will be maximum when $\theta = 45°$ and in this condition

$R = 4H \Rightarrow H = R/4$ (always)

because $R = 4H \cot\theta$ and $\theta = 45°$

So maximum height is 25% of maximum range.

It does not depends upon the velocity of projection.

11. (a) Range, $R = \frac{u^2 \sin 2\theta}{g}$

when $\theta = 45^o$, $R_{max} = \frac{u^2}{g}\sin 90^o = \frac{u^2}{g}$

when $\theta = 135^o$, $R_{max} = \frac{u^2}{g}\sin 270^o = \frac{-u^2}{g}$

Negative sign shows opposite direction.

12. (e) The man should point his rifle at a point higher than the target since the bullet suffers a vertically downward deflection $\left(y = \frac{1}{2}gt^2\right)$ due to gravity.

13. (b) In uniform circular motion, the magnitude of velocity and acceleration remains same, but due to change in direction of motion, the direction of velocity and acceleration changes. Also the centripetal acceleration is given by $a = \omega^2 r$.

14. (a) The body is able to move in a circular path due to centripetal force. The centripetal force in case of vehicle is provided by frictional force. Thus if the value of frictional force μmg is less than centripetal force, then it is not possible for a vehicle to take a turn and the body would overturn.

Thus condition for safe turning of vehicle is, $\mu mg \geq \frac{mv^2}{r}$.

15. (c) In circular motion the frictional force acting towards the centre of the horizontal circular path provides the centripetal force and avoid overturning of vehicle. Due to the change in direction of motion, velocity changes in circular motion.

16. (b) On an unbanked road, friction provides the necessary centripetal force $\frac{mv^2}{r} = \mu mg$ $\therefore v = \sqrt{\mu rg}$.

Thus with increase in friction, safe velocity limit also increases.

When the road is banked with angle of θ then its limiting velocity is given by $v = \sqrt{\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$.

Thus limiting velocity increase with banking of road.

17. (d) If the speed of a body is constant, all curved paths are possible.

In uniform circular motion a body has constant speed, but its direction keeps on changing, due to which it has non-zero acceleration.

18. (a) We know that $W = Fs\cos\theta$

in the circular motion if $\theta = 90^o$ then $W = 0$

19. (d) While moving along a circle, the body has a constant tendency to regain its natural straight line path.

This tendency gives rise to a force called centrifugal force. The centrifugal force does not act on the body in motion, the only force acting on the body in motion is centripetal force. The centrifugal force acts on the source of centripetal force to displace it radially outward from centre of the path.

20. (c) Centripetal force is defined from formula

$$F = \frac{mv^2}{r} \Rightarrow F \propto \frac{v^2}{r}$$

If v and r both are doubled then F also gets doubled.

21. (b) When automobile moves in circular path then reaction on inner wheel and outer wheel will be different.

$$R_{\text{inner}} = \frac{M}{2}\left[g - \frac{v^2h}{ra}\right] \text{ and } R_{\text{outer}} = \frac{M}{2}\left[g + \frac{v^2h}{ra}\right]$$

In critical condition $v_{\text{safe}} = \sqrt{\frac{gra}{h}}$

If v is equal or more that thus critical value then reaction on inner wheel becomes zero. So it leaves the ground first.

22. (c) For safe turn $\tan\theta \geq \frac{v^2}{rg}$.

It is clear that for safe turn v should be small and r should be large. Also bending angle from the vertical would increase with increase in velocity.

23. (a) When roads are not properly banked, force of friction between tyres and road provides partially the necessary centripetal force. This cause wear and tear of tyres.

24. (d) When the milk is churned centrifugal force acts on it outward and due to which cream in milk is separated from it.

25. (e) Due to earth's axial rotation, the speed of the trains relative to earth will be different and hence the centripetal forces on them will be different. Thus their effective weights $mg - \frac{mv^2}{r}$ and $mg + \frac{mv^2}{r}$ will be different. So they exert different pressure on the rails.

26. (d) Within a certain speed of the turn table the frictional force between the coin and the turn table supplies the necessary centripetal force required for circular motion. On further increase of speed, the frictional force cannot supply the necessary centripetal force. Therefore the coin flies off tangentially.

Motion In Two Dimension

SET Self Evaluation Test -3

1. Roads are banked on curves so that

(a) The speeding vehicles may not fall outwards

(b) The frictional force between the road and vehicle may be decreased

(c) The wear and tear of tyres may be avoided

(d) The weight of the vehicle may be decreased

2. In uniform circular motion

(a) Both velocity and acceleration are constant

(b) Acceleration and speed are constant but velocity changes

(c) Both acceleration and velocity changes

(d) Both acceleration and speed are constant

3. For a body moving in a circular path, a condition for no skidding if μ is the coefficient of friction, is

(a) $\frac{mv^2}{r} \le \mu mg$ (b) $\frac{mv^2}{r} \ge \mu mg$

(c) $\frac{v}{r} = \mu g$ (d) $\frac{mv^2}{r} = \mu mg$

4. A car is moving with a uniform speed on a level road. Inside the car there is a balloon filled with helium and attached to a piece of string tied to the floor. The string is observed to be vertical. The car now takes a left turn maintaining the speed on the level road. The balloon in the car will

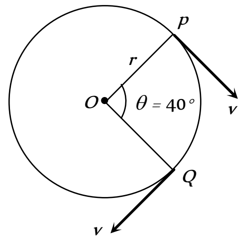
(a) Continue to remain vertical

(b) Burst while taking the curve

(c) Be thrown to the right side

(d) Be thrown to the left side

5. A particle is moving on a circular path of radius r with uniform velocity v. The change in velocity when the particle moves from P to Q is $(\angle POQ = 40^o)$



(a) $2v\cos 40^o$

(b) $2v\sin 40^o$

(c) $2v\sin 20^o$

(d) $2v\cos 20^o$

6. A body is revolving with a uniform speed v in a circle of radius r. The tangential acceleration is

(a) $\frac{v}{r}$ (b) $\frac{v^2}{r}$

(c) Zero (d) $\frac{v}{r^2}$

7. A particle does uniform circular motion in a horizontal plane. The radius of the circle is 20 *cm*. The centripetal force acting on the particle is 10 *N*. It's kinetic energy is

(a) 0.1 *J* (b) 0.2 *J*

(c) 2.0 *J* (d) 1.0 *J*

8. A body of mass m is suspended from a string of length l. What is minimum horizontal velocity that should be given to the body in its lowest position so that it may complete one full revolution in the vertical plane with the point of suspension as the centre of the circle

(a) $v = \sqrt{2lg}$ (b) $v = \sqrt{3lg}$

(c) $v = \sqrt{4lg}$ (d) $v = \sqrt{5lg}$

9. A particle moves with constant angular velocity in circular path of certain radius and is acted upon by a certain centripetal force F. If the angular velocity is doubled, keeping radius the same, the new force will be

(a) $2F$ (b) F^2

(c) $4F$ (d) $F/2$

10. In the above question, if the angular velocity is kept same but the radius of the path is halved, the new force will be

(a) $2F$ (b) F^2

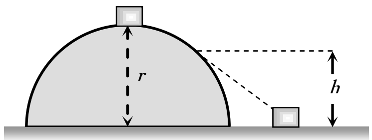
(c) $F/2$ (d) $F/4$

11. In above question, if the centripetal force F is kept constant but the angular velocity is doubled, the new radius of the path (original radius R) will be

(a) $2R$ (b) $R/2$

(c) $R/4$ (d) $4R$

12. A small body of mass m slides down from the top of a hemisphere of radius r. The surface of block and hemisphere are frictionless. The height at which the body lose contact with the surface of the sphere is



(a) $\frac{3}{2}r$ (b) $\frac{2}{3}r$

(c) $\frac{1}{2}gt^2$ (d) $\frac{v^2}{2g}$

13. A body of mass m kg is rotating in a vertical circle at the end of a string of length r metre. The difference in the kinetic energy at the top and the bottom of the circle is

(a) $\frac{mg}{r}$ (b) $\frac{2mg}{r}$

(c) $2mgr$ (d) mgr

14. A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'a' $meter/\sec^2$, then the resultant acceleration will be

(a) $\sqrt{\left\{\frac{v^2}{r^2}-a^2\right\}}$ (b) $\sqrt{\left\{\frac{v^4}{r^2}+a^2\right\}}$

(c) $\sqrt{\left\{\frac{v^4}{r^2}-a^2\right\}}$ (d) $\sqrt{\left\{\frac{v^2}{r^2}+a^2\right\}}$

15. A ball of mass 0.1 *kg* is suspended by a string. It is displaced through an angle of 60^o and left. When the ball passes through the mean position, the tension in the string is

(a) 19.6 *N* (b) 1.96 *N*

(c) 9.8 *N* (d) Zero

16. An aeroplane moving horizontally at a speed of 200 *m/s* and at a height of $8.0\times10^3\ m$ is to drop a bomb on a target. At what horizontal distance from the target should the bomb be released

(a) 7.234 *km* (b) 8.081 *km*

(c) 8.714 *km* (d) 9.124 *km*

17. A body is projected horizontally from a height with speed 20 *metres/sec*. What will be its speed after 5 *seconds* $(g=10\ metres/\sec^2)$

(a) 54 *metres/sec* (b) 20 *metres/sec*

(c) 50 *metres/sec* (d) 70 *metres/sec*

18. A man standing on the roof of a house of height h throws one particle vertically downwards and another particle horizontally with the same velocity u. The ratio of their velocities when they reach the earth's surface will be

(a) $\sqrt{2gh+u^2}:u$ (b) $1:2$

(c) $1:1$ (d) $\sqrt{2gh+u^2}:\sqrt{2gh}$

19. (A projectile projected at an angle 30^o from the horizontal has a range R. If the angle of projection at the same initial velocity be 60^o, then the range will be

(a) R (b) $2R$

(c) $R/2$ (d) R^2

20. At the highest point of the path of a projectile, its

(a) Kinetic energy is maximum

(b) Potential energy is minimum

(c) Kinetic energy is minimum

(d) Total energy is maximum

21. A cricket ball is hit at 30^o with the horizontal with kinetic energy K. The kinetic energy at the highest point is

(a) Zero (b) $K/4$

(c) $K/2$ (d) $3K/4$

22. A cannon on a level plane is aimed at an angle θ above the horizontal and a shell is fired with a muzzle velocity v_0 towards a vertical cliff a distance D away. Then the height from the bottom at which the shell strikes the side walls of the cliff is

(a) $D\sin\theta-\frac{gD^2}{2v_0^2\sin^2\theta}$ (b) $D\cos\theta-\frac{gD^2}{2v_0^2\cos^2\theta}$

(c) $D\tan\theta-\frac{gD^2}{2v_0^2\cos^2\theta}$ (d) $D\tan\theta-\frac{gD^2}{2v_0^2\sin^2\theta}$

23. A stone is projected from the ground with velocity 50 *m/s* at an angle of 30^o. It crosses a wall after 3 *sec*. How far beyond the wall the stone will strike the ground $(g=10\ m/\sec^2)$

(a) 90.2 *m* (b) 89.6 *m*

(c) 86.6 *m* (d) 70.2 *m*

24. A body of mass m is projected at an angle of 45^o with the horizontal. If air resistance is negligible, then total change in momentum when it strikes the ground is

(a) $2mv$ (b) $\sqrt{2}\,mv$

(c) mv (d) $mv/\sqrt{2}$

25. A ball of mass m is thrown vertically upwards. Another ball of mass $2m$ is thrown at an angle θ with the vertical. Both of them stay in air for same period of time. The heights attained by the two balls are in the ratio of

(a) 2 : 1 (b) $1:\cos\theta$

(c) 1 : 1 (d) $\cos\theta:1$

26. A particle is projected with a velocity v such that its range on the horizontal plane is twice the greatest height attained by it. The range of the projectile is (where g is acceleration due to gravity)

(a) $\frac{4v^2}{5g}$ (b) $\frac{4g}{5v^2}$

(c) $\frac{v^2}{g}$ (d) $\frac{4v^2}{\sqrt{5}g}$

Answers and Solutions

(SET -3)

1. (a) By doing so component of weight of vehicle provides centripetal force.

2. (c) Both changes in direction although their magnitudes remains constant.

3. (a) The value of frictional force should be equal or more than required centripetal force. *i.e.* $\mu mg\ge\frac{mv^2}{r}$

4. (d) Air outside the balloon is heavier so it will have more tendency to move towards right and will keep the balloon towards left side (Here in this question car is supposed to be air tight).

5. (c) Change in velocity = $2v\sin(\theta/2) = 2v\sin 20°$

6. (c) In uniform circular motion only centripetal acceleration works.

7. (d) $\frac{mv^2}{r} = 10 \Rightarrow \frac{1}{2}mv^2 = 10 \times \frac{r}{2} = 1\,J$

8. (d) For looping the loop minimum velocity at the lowest point should be $\sqrt{5gl}$.

9. (c) $F = m\omega^2 R \;\therefore\; F \propto \omega^2$ (m and R are constant)
If angular velocity is doubled force will becomes four times.

10. (c) $F = m\omega^2 R \;\therefore\; F \propto R$ (m and ω are constant)
If radius of the path is halved, then force will also become half.

11. (c) $F = m\omega^2 R \;\therefore\; R \propto \frac{1}{\omega^2}$ (m and F are constant)
If ω is doubled then radius will become 1/4 times *i.e.* $R/4$

12. (b)

13. (c) Difference in K.E. = Difference in P.E. = $2mgr$

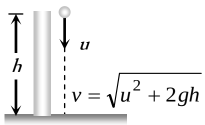
14. (b) $a_{\text{resultant}} = \sqrt{a_{\text{radial}}^2 + a_{\text{tangential}}^2} = \sqrt{\frac{v^4}{r^2} + a^2}$

15. (b) $T = mg + \frac{mv^2}{l} = mg + \frac{m}{l}[2gl(1-\cos\theta)]$
$= mg + 2mg(1-\cos 60°) = 2mg = 2 \times 0.1 \times 9.8 = 1.96N$

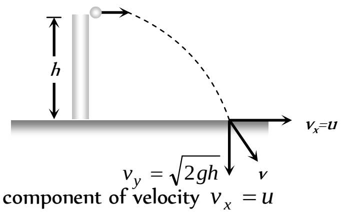
16. (b) Horizontal distance travelled by the bomb $S = u \times t$
$= 200 \times \sqrt{\frac{2h}{g}} = 200 \times \sqrt{\frac{2 \times 8 \times 10^3}{9.8}} = 8.081 km$

17. (a) Horizontal velocity $v_x = 20\,m/s$
Vertical velocity $v_y = u + gt = 0 + 10 \times 5 = 50\,m/\text{sec}$
Net velocity $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20)^2 + (50)^2} = 54\; m/s.$

18. (c) When particle thrown in vertical downward direction with velocity u then final velocity at the ground level



$v^2 = u^2 + 2gh \;\therefore\; v = \sqrt{u^2 + 2gh}$
Another particle is thrown horizontally with same velocity then*** at the surface of earth.



Horizontal component of velocity $v_x = u$
$\therefore$ Resultant velocity, $v = \sqrt{u^2 + 2gh}$
For both the particle final velocities when they reach the earth's surface are equal.

19. (a) For complementary angles of projection horizontal range is same.

20. (c) At the highest point of the path. Potential energy is maximum, so the kinetic energy will be minimum.

21. (d) Kinetic energy at the highest point

$$K' = K\cos^2\theta = K\cos^2 30 = K\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3K}{4}$$

22. (c) Equation of trajectory for oblique projectile motion

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Substituting $x = D$ and $u = v_0$

$$h = D\tan\theta - \frac{gD^2}{2u_0^2\cos^2\theta}.$$

23. (c) Total time of flight $= \frac{2u\sin\theta}{g} = \frac{2 \times 50 \times 1}{2 \times 10} = 5\,s$
Time to cross the wall = 3 *sec* (given)
Time in air after crossing the wall = (5 – 3) = 2 *sec*
$\therefore$ Distance travelled beyond the wall $= (u\cos\theta)t$
$= 50 \times \frac{\sqrt{3}}{2} \times 2 = 86.6\,m$

24. (b) Change in momentum= $2mv\sin\theta = 2mv\sin\frac{\pi}{4} = \sqrt{2}mv$

25. (c) The vertical components of velocity of both the balls will be same if they stay in air for the same period of time. Hence vertical height attained will be same.

26. (a) $R = 2H$ given
We know $R = 4H\cot\theta \Rightarrow \cot\theta = \frac{1}{2}$
From triangle we can say that $\sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$
$\therefore$ Range of projectile $R = \frac{2v^2\sin\theta\cos\theta}{g}$
$= \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$.